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Three essays on agricultural risk and insurance

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Three essays on agricultural risk and insurance

by

Li Zhang

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

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ABSTRACT

The general theme of this dissertation is agricultural risk and insurance in the United States. Chapter 2 examines welfare effects of the 2002 farm bill programs and yield insurance as well as their impacts on acreage decision of a representative Iowa farmer. Instead of measuring welfare using expected utility to capture farmers' preferences over risky alternatives, we apply recent advances in decision theory and use prospect theory to measure welfare changes due to government programs. The results indicate that there is no policy distortion to farmers' acreage decisions and farmers' willingness to pay per dollar of program cost is greatest for crop insurance. Given that farmers have crop insurance, the willingness to pay per dollar of program cost is much lower for loan deficiency payments, direct payments, and counter-cyclical payments. Chapter 3 develops a method for determining the aggregate risk of a book of business using hail insurance data. A spatial statistical approach is employed to measure the spatial correlation of hail loss cost. Monte Carlo simulation techniques are employed to simulate hail losses for a wide range of books of business. A regression model is estimated that captures the essence of the Monte Carlo simulation. This model can then be used to quickly estimate the degree of poolability of any given book of business. Chapter 4 turns to weather-based index contracts as alternative risk-management instruments in agriculture. A major concern associated with index contracts is basis risk. To address spatial basis risk, two spatial interpolation approaches, a geo-statistical approach and a Markov random field approach, are compared. The Markov random field approach is preferred because it has a smaller cross-validation prediction mean squared error. A temperature index insurance is presented based on interpolated data. The potential

performance of the proposed index insurance is investigated through historical analysis in contract years 1980 to 2005.

CHAPTER 1. GENERAL INTRODUCTION

1.1 Introduction

There is a widespread belief that the risk of crop losses cannot be effectively pooled by private insurance companies because of the systemic nature of risks inherent in crop insurance and because of asymmetric information problems, particularly adverse selection and moral hazard problems. Systemic risk in agriculture stems primarily from the impact of geographically extensive unfavorable weather events, such as droughts, floods or extreme temperatures, which induce significant correlation among individual yield losses, thereby defeating insurer efforts to pool risks across farms. Moral hazard occurs when the insured changes his behavior after purchasing the insurance so that the probability of receiving an indemnity increases. Adverse selection occurs when farmers have more information about the risk of loss than the insurer does. Adverse selection describes the situation where farmers who realize that their expected indemnity will exceed premium are more likely to buy insurance than those who don't. Moral hazard and adverse selection induce high transaction costs to insurers who ultimately must pass these expenses onto insurance purchasers by loading premium rates.

Despite the problems mentioned above, government supports crop insurance because it provides ex ante risk protection, rather than ex post forms of disaster assistance. However, as a mechanism for providing subsidies, government premium subsidies are often inefficient and associated with high social cost. The first topic of this dissertation, presented in Chapter 2, is about the welfare effects of 2002 farm bill programs and yield crop insurance as well as their impacts on acreage decision of a representative Iowa farmer who receives both farm

program benefits and crop insurance benefits. The 2002 farm bill contains three basic income support payments: counter-cyclical payments, loan deficiency payments, and direct payments. In addition, most acreage planted to crops supported by the farm bill programs also is insured under the U.S. crop insurance program. Therefore, there are a total of four government programs available for farmers to use against risks in agricultural production. The question addressed is whether U.S. farm programs provide too much protection, where “too much” is defined as low marginal willingness to pay by farmers for each unit of expected protection. Instead of measuring welfare using expected utility to capture farmers’ preferences over risky alternatives, we apply recent advances in decision theory and use prospect theory to measure welfare changes due to government programs. The representative farmer allocates acreage to corn, soybeans and alfalfa hay which are the three most widely grown crops in Iowa. Monte Carlo simulation technique is employed based on closed-form probability density functions of crop yields and futures prices. The farmer’s acreage decision is made by maximizing farmers’ expected profit, expected utility and expected value from prospect theory via a grid search method. Certainty equivalent returns (CER) are used to measure benefits of the programs. By comparing DPs only, CCPs only, LDPs only, or insurance only with no program at all, we get the willingness to pay for these programs respectively. By comparing all programs with removing insurance only, LDPs only, CCPs only, or DPs only, we get the willingness to accept measure for these programs respectively. Changes in optimal acreage, CER and expected profit allow calculation of the welfare changes and program efficiency.

The systemic nature of crop losses defeats insurers' effort to pool risks across space. The existence of private hail insurance market indicates that hail-caused yield loss risk can be made poolable. The essential difference between crop insurance and hail insurance lies in the level of risk dependence across space. The higher the level of risk dependence across space, the lower the risk poolability. Given a book of business, a question that insurers are extremely interested in is how large is systemic risk and what is the degree of poolability? To measure the degree of poolability of a book of business, people can apply stochastic simulation models of insurance indemnities to compute the variability of total indemnities paid. However, this procedure is complicated and time-consuming. In addition, the procedure needs to be repeated in order to evaluate risk poolability for each book of business. The objective of the second topic of this dissertation, Chapter 3, is to develop a method to measure risk poolability of any specific book of business quickly by simply knowing a few key statistics of the given book of business. The method is developed using hail insurance data. The development proceeds as follows. First, a spatial variogram is estimated and a theoretical model of loss cost resulting from hail damage in Iowa is fit to the empirical data. Thus, we explicitly model hail loss using a spatial statistical approach. Next, hail losses are simulated for a wide range of books of business using Monte Carlo simulation. And finally, a regression model is estimated that captures the essence of the Monte Carlo simulation. This model can then be used to quickly estimate the degree of poolability of any given book of business.

Concerns over the systemic nature of crop losses and costs of insuring farm-level crop yield have prompted increased interest in weather-based index contracts as alternative risk-management instrument in agricultural production. The underlying weather index is

usually a weather variable or a function of multiple weather variables accumulated over a period of time. There are many advantages of weather index contracts over the traditional individual-yield and area-yield crop insurance. First, individual who use an index contract should be unable to influence the outcome that determines payment from the contract. Monitoring needs are reduced, which lowers transaction costs. Second, the indemnity structure is not directly tied to actual crop yield. This saves lost adjustment costs and eliminates the possibility of moral hazard. Adverse selection is minimized or eliminated because premium calculation is based on objective weather events which are independent of participation of producers in the program. Last, aggregating production risks across space may reduce the idiosyncratic risk in the aggregate portfolio, the insurer can then hedge the systemic weather risk using weather derivatives via the Chicago Mercantile Exchange and over-the-counter security markets.

The major concern associated with index contract is basis risk. There are two layers of weather basis risk. The first layer, or the spatial basis risk, refers to the fact the weather index value defined at a weather station may not be the same as the realized weather index value at a specified location. The second layer, or the technological basis risk, refers to the fact that the underlying weather index is an imperfect hedge against risk exposure even if the underlying index and exposure being hedged correspond to the same location. As a result, producer may not receive an indemnity even if he/she suffers a production loss, or alternatively, may receive an indemnity even though no loss has occurred. Basis risk has been cited as a primary concern for the implementation of weather hedges in many studies. However, most of these studies put emphasis on solving the second layer of basis risk, leaving spatial basis risk as still an open question. As Richards et al. (2004) mentioned,

economic research can do little to remedy the spatial basis risk problem. There are two main approaches to address spatial dependence problems. One is the standard, well-developed geo-statistical approach. The other is the less-developed Markov random field (MRF) approach. Both the geo-statistical approach and the MRF approach have strengths and weaknesses in terms of operational and data-analytic aspects. The third topic of this dissertation, presented in Chapter 4, focuses on the spatial basis risk in the implementation of weather-based index contract. A temperature index insurance contract is designed to provide protection to corn growers in Iowa. The contract is essentially an exotic call option on the temperature index, cooling degree days, accumulated during the summer season (ACDD). To address the spatial basis risk in implementing this temperature index insurance, Both the geo-statistical model and the MRF model in addition to a naive multiple linear regression model which assume no underlying spatial correlation of ACDD are fitted to the data. MRF approach is preferred in the sense of smaller cross-validation prediction mean squared error and the fact that MRF approach promises straightforward extension to multiple years of data in model estimation. The insurance policy is rated using Monte Carlo simulation assuming that ACDD at a given location follows normal distribution. ACDD is interpolated for locations where recorded ACDD data is not available through MRF prediction approach. Potential performance of the proposed insurance policy is investigated through historical analysis in contract years 1980 to 2005.

1.2 Organization of the dissertation

The dissertation is organized into five chapters. The first chapter provides a general introduction to the three topics discussed in this dissertation which are presented in chapter 2,

chapter 3 and chapter 4, respectively. The three topics are related with regard to the general theme of crop risks and insurance. However, the three topics also stand alone with each topic addressing a different component of the general theme. Chapter 2 examines the welfare effects of 2002 farm bill programs and yield crop insurance as well as their impacts on acreage decision of a representative Iowa farmer who receives both farm program benefits and crop insurance benefits. Chapter 3 develops a method to measure risk poolability of any specific book of business quickly by simply knowing a few key statistics of the given book of business of which the insured risks are spatially correlated. Chapter 4 turns to weather-based index insurance contracts as alternative risk-management instruments in agricultural protection. The study focuses on reducing basis risk which is the major concern in the implementation of weather index insurance contracts. Chapter 5 gives a general conclusion to the research findings on the three topics discussed in this dissertation.

CHAPTER 2. USING PROSPECT THEORY TO EVALUATE STOCHASTIC FARM PROGRAM PAYMENTS

Abstract

Passage of the 2002 farm bill together with the Agricultural Risk Protection Act passed in 2000 brought increased protection for farm income against both adverse price movement and crop losses. The farm bill contains three basic income support payments: counter-cyclical payments, loan deficiency payments, and direct payments. In addition, most acreage planted to crops supported by the farm bill programs also is insured under the U.S. crop insurance program. Many observers question the efficiency of these programs because of the high level of protection. The question addressed by this research is whether U.S. farm programs provide too much protection, where “too much” is defined as low marginal willingness to pay by farmers for each unit of expected protection. This paper examines farmers’ willingness to pay for these programs as well as their impacts on acreage decisions. Instead of measuring welfare using expected utility to capture farmers’ preferences over risky alternatives, we apply recent advances in decision theory and use prospect theory to measure welfare changes due to government programs. The results indicate that of these four programs, farmers’ willingness to pay per dollar of program cost is greatest for crop insurance. Given that farmers have crop insurance, the willingness to pay per dollar cost is much lower for loan deficiency payments, direct payments, and counter-cyclical payments. The efficiency measures of these government programs are low because of three reasons. First, the sum of the decision weights is 0.74, which implies that 26% of the expected value is eliminated at the very beginning. Second, the payments from any individual government

program is small which only move the farmer from the state of loss to another state of loss where loss aversion coefficient shrinks the WTP for programs by a factor of 1/2.25. And last, the payments from these government programs are made not only when there is a loss but also when there is a gain. The change in expected value is lower when many of payments are made in the gains part relative to the change in expected value when all the payments are made in the loss part. As far as the distortionary impacts of the programs are concerned, we find that under either harsh or no yield penalties for planting soybeans after soybeans or corn after corn, there is no policy distortion to farmers' acreage decisions. Therefore, government programs act as lump-sum transfers to Iowa farmers with regard to their acreage decisions. WTA valuations are higher than WTP valuations of the individual programs suggest that there is an endowment effect. It leads to more pain to farmers to be deprived of those government programs when farmers originally have access to them than joy to farmers by providing farm programs when they are originally not available to farmers.

2.1 Introduction

Passage of the 2002 farm bill along with the Agricultural Risk Protection Act passed in 2000 brought increased protection of farm income against both adverse price movement and crop losses. The farm bill contains three basic income support payments: counter-cyclical payments (CCPs), loan deficiency payments (LDPs), and direct payments (DPs). In addition, most acreage planted to crops supported by the farm bill programs are also insured under the U.S. crop insurance (CI) program. Thus, there are four government programs that farmers can use for income support and risk reduction.

DPs are fixed payments to farmers made based on a producer's historical base production. The DP rate is a fixed payment amount per unit of base production. CCPs give additional payments to farmers when market prices fall below the effective target price of crops. CCP payments are based on historical production levels also. The CCP payment rate is not fixed like the DP rate, but depends on the 12-month marketing year average for each eligible crop. Because DPs and CCPs depend on base acres and base yields, they are decoupled from a farmer's current production decision. LDPs are made when the posted county price is below the loan rate for the county. Farmers can choose which day they take their LDPs. This payment is made based on the producer's current production instead of historical production. Therefore, LDPs are coupled. The payment rate is determined by the national loan rate and posted county prices on the chosen execution day. While DPs are lump-sum payments, CCPs and LDPs provide price insurance.

The U.S. crop insurance program provides farmers the opportunity to insure against unexpected revenue or yield declines. In this analysis, we focus on yield insurance. A payment from yield insurance is made to farmers when actual production falls below the insured yield. Lost bushels are compensated at the insured price.

The large amount of support given to U.S. farmers creates both domestic and foreign pressures to move U.S. agricultural toward a greater market orientation. Large U.S. budget deficits also create reform pressure on government to reduce the magnitudes of domestic interventions in agricultural markets. Taking Iowa farmers as an example, this paper provides an analysis of the welfare impacts on farmers from these government support programs and the impacts of these farm programs on a farmer's acreage decisions.

Much theoretical and empirical work has been done on the effects of crop insurance on production. (e.g., Ahsan, Ali and Kurian; Nelson and Loehman; Chambers and Quiggin; Wu; Keeton, Skees and Long; Robinsn; Yong et al). Yet, only limited research has been conducted investigating the effects of DPs, LDPs and CCPs. One exception is a study by Hennessy (1998) which investigates the wealth, coupling and income-insurance effects of alterations in US agricultural policies that occurred in the 1996 farm bill. Hennessy examined the nitrogen decisions of a representative farmer in Iowa who produces 400 acres of continuous corn. He analyzes the welfare effects of farm program using expected utility theory with a utility function which accommodates both constant absolute risk aversion (CARA) and decreasing absolute risk aversion (DARA). His conclusion is that income support policies that are assumed to be decoupled are not, in fact, completely decoupled because of both wealth and insurance effects.

Another exception is a study by Hennessy, Babcock and Hayes (1997) which investigate the budgetary and producer welfare effects of revenue insurance and the 1990 farm bill when LDPs are the only farm program. They showed that the 1990 farm program is not efficient and producers' welfare can be enhanced by simply giving decoupled lump-sum payments. They also showed that a 75% revenue insurance scheme would result in a very large reduction in government outlays and would be more efficient than the 1990 farm program. They measure efficiency in terms of farmer benefit per dollar cost of program protection. To measure producers' welfare, expected utility theory framework is applied in their study.

In this research, our primary goal is to examine the welfare effects of the 2002 farm bill programs and yield crop insurance as well as their impacts on the acreage decisions of a

representative Iowa farmer who receives both farm program benefits and crop insurance benefits. Instead of measuring welfare using expected utility to capture farmers' preferences over risky alternatives, we apply recent advances in decision theory and use prospect theory to measure welfare changes due to government programs. The representative farmer allocates acreage to corn, soybeans and alfalfa hay. We limit the choice set to these three crops because they are the three most widely grown crops in Iowa. We include alfalfa hay in our analysis because we want to include a non-program crop that a farmer can move into if they find program crops are not profitable. With three crop choices, farmers' acreage decision problem becomes a two-decision-variable problem: how many acres allocated to corn and how many acres allocated to soybeans, with whatever is left being allocated to alfalfa hay. We assume that a farmer's income comes completely from crop production and no other source. Traditional welfare analyses of farm programs use "Harberger Triangle" measures (Johnson, 1965). Such methods are less appropriate given the form of the policy interventions that we have today, because the amount of payments depends on the realizations of prices and yields. In this research, we do an ex ante stochastic analysis to measure the distortion and welfare impacts of U.S. farm policies.

Monte Carlo simulations of program payments and farm income are performed based on closed-form probability density functions of crop yields and futures prices. We assume that crop yields follow beta distributions and prices follow lognormal distributions. Correlations among yields and prices are derived from historical data. Welfare changes due to farm programs are measured by ex ante willingness to pay to obtain farm programs and ex ante willingness to accept a loss of farm programs. Calculation of these welfare changes in proposed theory requires specification of an appropriate preference function to allow an

endowment effect. After a series of calibrations, we find that the value function of Kahneman and Tversky (1979) together with the compound-invariant weighting function of Prelec (1998) can give a globally consistent characterization of farmers' preferences over risky alternatives. A set of reasonable parameter is obtained for this study by taking guidance from other studies. To model farmers' price expectations, we follow Hoffman (2005) and apply the futures price forecasting model to forecast the season average price (SAP) of U.S. corn and soybeans. A straightforward extension of Hoffman's approach allows price volatilities to be obtained for use in the Monte Carlo analysis. Expected posted county prices and selling price can be obtained from expected season average prices.

With calibrated preferences and simulated data, the farmer's acreage decision for corn and soybeans respectively is made by maximizing farmers' expected profit, expected utility and expected value from prospect theory via a grid search method. Certainty equivalent returns (CER) are used to measure benefits of the programs. By comparing DPs only, CCPs only, LDPs only, or insurance only with no program at all, we get the willingness to pay for these programs respectively. By comparing all programs with removing insurance only, LDPs only, CCPs only, or DPs only, we get the willingness to accept measure for these programs respectively. Changes in optimal acreage, CER and expected profit allow calculation of the welfare changes and program efficiency.

2.2 Methodology

Monte Carlo simulations are performed based on closed-form probability density functions of crop yields and futures prices. Although there are no firm guidelines about the appropriate functional forms to use to model stochastic yields, the literature does suggest that

selected functional forms should be somewhat flexible. Day (1965) showed that crop yields are skewed and found the beta distribution to be an appropriate functional form for parametric estimation. Babcock and Blackmer (1992), Borges and Thurman (1994), Babcock and Hennessy (1996), and Coble et al. (1996) have all used beta distributions in their applied work. Here, in this analysis, we assume that crop yields follow beta distributions. The density function is of the form:

$$p(y) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \cdot \frac{(y-a)^{p-1}(b-y)^{q-1}}{(b-a)^{p+q-1}} \quad (a \leq y \leq b)$$

Where $p > 0$, $q > 0$ are shape parameters and a and b are the minimum and maximum possible yield respectively.

We assume that crop futures prices for each contract month follow lognormal distributions, for which only estimates of mean and volatility are required to define the distribution. We model the acreage decisions of a farmer for crop year 2006. Harvested production is sold in the 2006 marketing year. For marketing year 2006, contract months for corn futures are September and December of 2006, March, May and July of 2007; contract months for soybeans futures are September and November of 2006, January, March, May, July and August of 2007. The farmer allocates land to corn, soybeans and alfalfa hay.

The distributions of crop yields and crop prices are outlined in Table 2.1. There are sixteen random variables in total: three yield variables, five corn futures prices, seven soybeans futures prices and the alfalfa hay price. The mean yields for corn and soybeans are assumed at the 2006 expected yields levels in Story County, Iowa which can be obtained from USDA-RMA. The standard deviations of corn and soybean yields are calibrated to be consistent with a 65% coverage level of crop insurance in Boone County, Iowa. We assume a

Table 2.1. Distribution of crop yields and (futures) prices

Variable	Distribution	Mean	Standard Deviation Or Volatility
Corn Yield	Beta	169.30 (bu/acre)	50.70 (bu/acre)
Soybean Yield	Beta	45.60 (bu/acre)	13.80 (bu/acre)
Hay Yield	Beta	4.00 (ton/acre)	0.53 (ton/acre)
Hay Price	Lognormal	65.00(\$/ton)	12.35(\$/ton)
Corn Contract Months			
September	Lognormal	2.44(\$/bu)	0.20
December	Lognormal	2.54(\$/bu)	0.22
March	Lognormal	2.60(\$/bu)	0.23
May	Lognormal	2.64(\$/bu)	0.24
July	Lognormal	2.67(\$/bu)	0.25
Soybeans Contract Months			
September	Lognormal	6.14(\$/bu)	0.12
November	Lognormal	6.21(\$/bu)	0.22
January	Lognormal	6.25(\$/bu)	0.23
March	Lognormal	6.28(\$/bu)	0.23
May	Lognormal	6.35(\$/bu)	0.24
July	Lognormal	6.34(\$/bu)	0.25
August	Lognormal	6.34(\$/bu)	0.26

well-managed farm so that the mean yield of alfalfa hay is four tons per acre. The standard deviation of yield is derived from historical hay yields from 1981 to 2004 in Boone County, Iowa. The mean of the hay price is the expected harvesting price at planting time. The standard deviation of the price of hay is derived from the historical price of hay from 1981 to 2004 in Boone County, Iowa. We use the CBOT futures prices on Jan 27th, 2006 as the mean price and calculate the volatility of each contract month futures price so that the option premium for the at-the-money put option of each contract month is equal to the corresponding CBOT option premium on Jan. 27th, 2006.

In implementing the Monte Carlo procedure, it is important that the method incorporate the correlation among those random variables. The correlation matrix is derived by analyzing historical data. Historical yields of the three crops and the historical hay price data for Boone County, Iowa for years 1990 to 2004 can be obtained from the USDA-NASS.

After detrending, we can calculate the percentage deviations of yield of each crop by taking the ratio of detrended yields to the corresponding mean yield. To obtain the hay price's percentage differences, we calculate the mean, subtract it from the original price series and take the ratio of demeaned prices over the mean price. CBOT daily futures price of each contract month and realized daily price of each contract month are also available for years 1990 to 2004. After calculating the average CBOT futures price and the average realized price for each contract month for each of these years, we then take the differences of these two average measures for each month to get a series of price differences. Now, we have thirteen series of price differences and three series of yield percentage deviations. The sample correlation matrix of these data is reported in Table 2.2.

After obtaining the correlation matrix, 5000 standard-normal deviates for the sixteen random variables are generated. Cholesky decomposition of the covariance matrix was used to impose the desired level of correlation. Then, draws of crop yields, hay prices and contract month futures prices for corn and soybeans can be obtained by transforming these correlated normal deviates because transformation itself will have little impact on the degree of correlation among the random variables. We know that alfalfa hay is a no-subsidy crop, so its yield and price draws are used directly in this analysis. Yield draws of corn and soybeans are also used directly. However, for corn and soybeans, the prices we need in the analysis are the season average price (SAP), the selling price (P), and the posted county price (PCP). We need a method to predict these prices from the draws of future prices. A crop's SAP is a 12-month marketing year weighted average price. We take this price as the basic price for our analysis because it is the average price

Table 2.2. Correlations of crop yields and (futures) prices

	Corn Contract Months				Soybeans Contract Months								Yields		Price	
	SEP	DEC	MAR	MAY	JUL	SEP	NOV	JAN	MAR	MAY	JUL	AUG	Corn	Soybeans	Hay	Hay
Corn Contract Months																
SEP	1	0.72	0.73	0.63	0.49	0.91	0.65	0.66	0.62	0.61	0.50	-0.21	-0.21	-0.12	-0.21	0.23
DEC	0.72	1	0.93	0.85	0.75	0.69	0.76	0.76	0.57	0.54	0.47	-0.35	-0.44	-0.32	-0.26	0.09
MAR	0.73	0.93	1	0.95	0.83	0.64	0.73	0.77	0.66	0.66	0.54	-0.36	-0.24	-0.19	0.01	0.01
MAY	0.63	0.85	0.95	1	0.93	0.50	0.62	0.69	0.56	0.61	0.52	-0.17	-0.05	0.02	0.23	-0.07
JUL	0.49	0.75	0.83	0.93	1	0.32	0.43	0.52	0.39	0.43	0.50	0.14	0.14	0.22	0.33	-0.04
Soybeans Contract Months																
SEP	0.91	0.69	0.64	0.50	0.32	1	0.77	0.76	0.70	0.67	0.55	-0.35	-0.35	-0.43	-0.37	0.19
NOV	0.65	0.76	0.73	0.62	0.43	0.77	1	0.97	0.87	0.86	0.77	-0.56	-0.30	-0.48	-0.23	0.12
JAN	0.66	0.76	0.77	0.69	0.52	0.76	0.97	1	0.93	0.92	0.85	-0.45	-0.28	-0.46	-0.17	0.09
MAR	0.62	0.57	0.66	0.56	0.39	0.70	0.87	0.93	1	0.98	0.90	-0.44	-0.15	-0.42	-0.06	0.09
MAY	0.61	0.54	0.66	0.61	0.43	0.67	0.86	0.92	0.98	1	0.89	-0.41	-0.08	-0.32	0.03	0.07
JUL	0.50	0.47	0.54	0.52	0.50	0.55	0.77	0.85	0.90	0.89	1	-0.10	0.10	-0.21	0.08	0.11
AUG	-0.21	-0.35	-0.36	-0.17	0.14	-0.35	-0.56	-0.45	-0.44	-0.41	-0.10	1	0.43	0.53	0.34	-0.04
Yields																
Corn	-0.21	-0.44	-0.24	-0.05	0.14	-0.35	-0.30	-0.28	-0.15	-0.08	0.10	0.43	1	0.76	0.77	-0.15
Soybeans	-0.12	-0.32	-0.19	0.02	0.22	-0.43	-0.48	-0.46	-0.42	-0.32	-0.21	0.53	0.76	1	0.56	0.06
Hay	-0.21	-0.26	0.01	0.23	0.33	-0.37	-0.23	-0.17	-0.06	0.03	0.08	0.34	0.77	0.56	1	-0.37
Price																
Hay	0.23	0.09	0.01	-0.07	-0.04	0.19	0.12	0.09	0.09	0.07	0.11	-0.04	-0.15	0.06	-0.37	1

at which farmers sell their crops.

We now turn to the problem of how to estimate our farmer's expectation of SAP for corn and soybeans. We first draw contract month futures prices from their respective distributions as shown in Table 2.1. We can use linear interpolation to obtain predicted values for non-contract months' futures prices. Because the SAP is a 12-month marketing year weighted average price, we need to know what the weight is for each month of a marketing year. Looking at the historical percentage of sales of crop by month in USDA's annual Agricultural Prices Summary from year 1994 to 2004, we see no obvious shift in the sales percentage pattern over this time period. Therefore simple averages of those monthly sales percentages across years are taken as the weights. The historical and average percentage of sales of corn and soybeans for each month of a marketing year are shown in table 2.3 and Table 2.4, respectively. Multiplying the twelve monthly futures prices by the twelve monthly weights mentioned above, we can derive the expected SAP of crops for years 1995 to 2004. Expected SAP is used to determine expected selling price (P) and expected posted county price (PCP).

To see the bias of our expected SAP using futures prices from the actual SAP, we compare the estimated SAP and the actual NASS survey SAP from 1995 to 2004. The actual NASS survey SAP can be obtained from annual Agricultural Prices Summary. On average there is an upward bias of \$0.153 for corn and \$0.367 for soybeans, namely,

$$SAP_c = E(SAP_c) - 0.153$$

$$SAP_s = E(SAP_s) - 0.367$$

Table 2.3. Historical monthly sale percentage of corn in a marketing year

YEAR	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG
1994-1995	5.0	11.2	12.2	8.5	16.3	7.9	7.7	5.3	5.8	6.2	6.9	7.0
1995-1996	8.1	17.1	12.9	8.0	17.4	8.3	8.5	6.2	4.3	3.3	3.4	2.5
1996-1997	4.0	11.4	13.2	8.0	15.1	9.8	8.0	6.1	4.8	5.6	6.7	7.3
1997-1998	6.2	14.1	11.3	8.1	14.8	6.7	7.4	5.2	5.3	7.4	6.2	7.3
1998-1999	7.5	13.9	10.3	7.2	12.7	7.6	9.0	4.8	4.3	5.8	7.4	9.5
1999-2000	9.5	14.2	8.3	6.9	17.2	6.7	8.0	5.5	4.2	4.3	6.1	9.1
2000-2001	11.3	15.8	8.6	6.8	15.0	5.3	6.4	4.9	4.6	5.4	8.2	7.7
2001-2002	7.8	11.0	13.3	6.6	12.5	6.0	6.2	6.0	6.3	7.5	8.8	8.0
2002-2003	7.1	14.0	13.9	8.2	12.6	6.1	6.7	6.5	6.5	5.3	6.0	7.1
2003-2004	7.0	15.6	11.6	8.7	16.7	7.2	8.0	5.7	3.3	5.3	5.6	5.3
Average	7.4	13.8	11.6	7.7	15.0	7.2	7.6	5.6	4.9	5.6	6.5	7.1

Table 2.4. Historical monthly sale percentage of soybeans in a marketing year

YEAR	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG
1994-1995	7.0	20.6	10.0	7.7	12.7	7.9	8.8	5.0	5.5	4.4	6.0	4.4
1995-1996	8.1	26.9	8.5	7.2	15.2	6.1	6.3	5.8	3.8	3.4	4.7	4.0
1996-1997	3.9	21.0	10.4	7.2	17.8	8.3	7.9	6.1	4.4	3.9	4.3	4.8
1997-1998	6.3	22.8	8.9	8.8	12.3	7.2	6.0	6.1	5.2	6.3	5.6	4.5
1998-1999	9.0	22.6	8.4	6.1	10.2	6.3	8.2	4.9	4.4	5.9	6.4	7.6
1999-2000	10.0	23.9	6.1	6.2	15.3	6.0	7.4	4.8	4.0	3.7	5.9	6.7
2000-2001	8.7	23.1	8.2	7.1	14.0	6.6	6.4	5.1	6.5	5.3	5.3	3.7
2001-2002	4.5	21.9	10.9	8.3	16.0	6.6	9.1	5.4	4.2	4.1	5.0	4.0
2002-2003	5.8	21.2	10.3	9.3	14.4	8.3	6.6	7.5	4.7	3.8	4.6	3.6
2003-2004	8.8	29.5	9.4	7.9	16.9	6.6	6.1	3.5	2.4	2.2	3.4	3.2
Average	7.2	23.4	9.1	7.6	14.5	7.0	7.3	5.4	4.5	4.3	5.1	4.7

For corn and soybeans, PCP and P are functions of SAP. According to Babcock and Hart (2005), the wedge between SAP and PCP in Boone County, Iowa is \$0.26 for corn and \$0.56 for soybeans. Thus, we have,

$$PCP_c = SAP_c - 0.26$$

$$PCP_s = SAP_s - 0.56$$

Data covering the last five years of price basis for Boone County is available on the CARD website. The weighted average crop basis from CBOT prices is computed as \$0.314 for corn and \$0.276 for soybeans in Boone County, Iowa. Here, we use the same weights as

those in estimating SAP. The selling price of the crop is equal to CBOT prices minus the weighted average Boone crop basis:

$$P_c = \text{CBOT}_c - 0.314 = \text{SAP}_c + 0.153 - 0.314 = \text{SAP}_c - 0.161$$

$$P_s = \text{CBOT}_s - 0.276 = \text{SAP}_s + 0.367 - 0.276 = \text{SAP}_s + 0.091$$

One thing needs to be noted is that there is a rotation effect in planting crops because of soil erosion, lack of pest control and nitrogen changes in availability. According to Hennessy (1998) and ISU (2002), if corn is followed by another year of corn, corn yields will on average decrease by 10% in Iowa. If soybeans are followed by another year of soybeans, soybeans yields will on average decrease by 12.5%.

The mathematical problem is to choose acres for corn, soybeans and alfalfa hay to maximize expected profit and expected utility of the farmer. The total profit from three crops is:

$$\Pi = \sum_{i=1}^3 (A_i(p_i y_i - c_i) + g p_i), \quad i = 1, 2, 3$$

$i=1, 2$ and 3 denotes corn, soybeans and alfalfa hay, respectively; p_i is the price of crop i ; y_i is the yield of crop i ; c_i is the variable cost of crop i . Thus $A_i(p_i y_i - c_i)$ is the net profit from crop i , where A_1 is the percentage of land planted to corn, A_2 is the percentage of land planted to soybeans and A_3 is the percentage of land planted to hay which is equal to $1 - A_1 - A_2$. $g p_i$ is the income of crop i coming from government programs, which is the sum of the income from DP , CCP , LDP and crop insurance (CI) program respectively. Each of these programs is defined as follows:

$$DP_i = (DP_{rate}_i) \times (DP_{base\ acre}_i) \times (DP_{base\ yield}_i) \times 0.85;$$

Direct payment of crop i is a fixed payment based on 85% of a farmer's historical production base of crop i . DP rate $_i$ is a fixed payment amount per unit of base production (DP base acre \times DP base yield) for crop i .

$$CCP_i = \max (CCP \text{ rate}_i, 0) \times (CCP \text{ base acre}_i) \times (CCP \text{ base yield}_i) \times 0.85;$$

where $CCP \text{ rate}_i = (\text{target price}_i - DP \text{ rate}_i) - \max(\text{national loan rate}_i, SAP_i)$.

CCP payments provide additional payments to producers when market prices fall below the trigger price. The payments are made based on historical production level. But, the CCP payment rate per yield unit of crop is not fixed, which depends on the target price, the direct payment rate, the national loan rate and the 12-month marketing year average price of crop i .

$$LDP_i = \max (LDP \text{ rate}_i, 0) \times (A_i) \times (\text{actual yield}_i);$$

where $LDP \text{ rate}_i = \text{county loan rate}_i - \text{posted county price}$. LDP payments are made when the posted county price is lower than the loan rate. LDP payments are based on the actual production level rather than base production as with CCP and DP . Thus LDP is referred to a coupled payment, where CCP and DP are decoupled payments.

$$CI_i = \max (y_i^{ins} - y_i, 0) \times p_i^{ins} - \text{premium};$$

where premium = $E (\max (y_i^{ins} - y_i, 0) \times p_i^{ins}) \times (1 - \text{subsidy rate})$.

We assume that this farmer purchases yield insurance, which is provided by the APH (Actual Production History) crop insurance product. Insurance payments are made whenever actual yield falls below the insured yield level. The premium used here is set at actuarially fair levels. Subsidy rate is the portion of the premium that the government pays.

First Monte Carlo integration is used to obtain expected profit and expected utility of farmers. These solutions provide a base case from which to evaluate the effects of moving to Prospect Theory. Given Monte Carlo draws of prices and yields for corn, soybeans and hay, a grid search method is used to find the optimal acreage level that maximize the expected profit and the expected utility. We are primarily interested in the impacts of government programs on farmers' acreage decisions, so we assume farmers' input decisions like machinery or fertilizer usage are made independently of government programs at technology-efficient levels.

We begin by finding the optimal acreage using the expected utility theory framework by choosing acres for corn and soybeans in the baseline model (without government programs). Farmers' willingness to pay for government programs are represented by the change in certainty equivalent returns (CER) which can be easily calculated according to the specification of CARA utility. The risk premium ratio is simply the ratio of difference between expected profit and CER to the standard deviation of profit. The results are presented in Table 2.5.

We see from the table that as the risk aversion coefficient γ increases, the risk premium ratio increases, but that acreage allocation remains fixed at 50-50 rotation until the risk premium ratio moves above 41.54%, which is likely larger than that of the vast majority of commercial corn and soybean farmers. Thus we can conclude that for farmers with reasonable risk aversion level, alfalfa hay will not enter into solution of acreage decision problems. This is not surprising because both corn and soybean have higher expected crop profit than does hay. Furthermore, hay does not receive subsidies like corn and

Table 2.5. Optimal acreage allocations with a CARA utility function

Risk-aversion coefficient	$\gamma = 0.0001$	$\gamma = 0.001$	$\gamma = 0.005$	$\gamma = 0.007$	$\gamma = 0.008$	$\gamma = 0.009$
Profit maximizing corn and soybeans acreage shares	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)
Utility maximizing corn and soybeans acreage shares	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)	(0.42, 0.5)
Expected profit under expected profit maximization	195.79	195.79	195.79	195.79	195.79	195.79
Standard Deviation of profit	102.55	102.55	102.55	102.55	102.55	102.55
Expected utility under expected utility maximization	-0.98	-0.83	-0.43	-0.32	-0.28	-0.25
Certainty Equivalent Returns	194.89	190.56	170.95	161.66	157.21	153.19
Ratio of risk premium and standard deviation of profit	0.88%	5.10%	24.22%	33.27%	37.61%	41.54%

soybeans, hence the attractiveness of corn and soybeans will only increase once farm programs are taken into account.

2.3 Farmers' preferences over risk and uncertainty

In order to measure the welfare impacts of those government programs on farmers, we need to know farmers' preferences over risk and uncertainty first. Possible choices are risk-neutral preferences, expected utility theory and prospect theory.

2.3.1 Risk-neutral preferences

Given risk-neutral preferences, maximizing farmers' expected utility is equivalent to maximizing their expected profit. The advantage of choosing risk-neutral preference is that it is simple and easy to implement and we don't need to arbitrarily impose a curvature on farmers' preference. But there is ample evidence from empirical experiments examining choices over lotteries that people are not risk-neutral. For example, Selten, Reinhard, Abdolkarim Sadrieh, and Klaus Abbink (1995) found significant deviations from risk neutral behavior with binary lottery payoffs.

2.3.1 Expected utility theory

Expected utility theory has dominated the main normative and descriptive literature in decision making under risk. This is also true in the analysis of input and production decision-making in agriculture. (e.g., Hennessy (1998); Chavas and Holt (1990) and (1996); Ramaswami (1992); Collender and Zilberman (1985); Feder and Gershon (1980); Babcock and Hayes (2001)).

Within the expected utility theory framework, the only explanation for risk aversion is that the utility function for wealth is concave. People have lower marginal utility for additional wealth when he is wealthy than he is poor. Rabin (2000) provides a theorem showing that expected-utility theory is an utterly implausible explanation for appreciable risk aversion over small and modest stakes. Within expected utility theory framework, for any concave utility function, even very little risk aversion over small and modest stakes implies an absurd degree of risk aversion over large stakes. Neilson and Winter (2001) investigate whether a single constant relative risk aversion (CRRA) utility function can fit data for both small- and large-scale risks in both wage-fatality risk tradeoff data and portfolio choice data. They show that the coefficients of constant relative risk aversion compatible with wage-fatality risk premium data are smaller than the coefficients compatible with portfolio choice data, suggesting that a utility function used to evaluate large-stakes risks are less risk averse than those used to evaluate small- or moderate-stakes risks. This study lends further support to the contention that an expected utility preference representation with a single utility function is unable to describe choices over both small- or moderate-stakes risks and large-stakes risks.

Indeed, what is empirically the most firmly established feature of risk preferences, loss aversion, is a departure from expected-utility theory that provides a direct explanation for small and modest-scale risk aversion. Loss aversion says that people are significantly more averse to losses relative to the status quo than they are attracted by gains, and more generally that people's utilities are determined by changes in wealth rather than absolute levels (Rabin, 2000).

2.3.3 Prospect theory

Over the past 30 years, the expected utility theory framework has come under attack on a number of different fronts. First, Kahneman and Tversky (1992) argue that people's utilities are determined by changes in wealth rather than absolute levels. The traditional carrier-of-value in the utility function, the individual's final wealth position, is not appropriate. Second, Kahneman and Tversky (1992) demonstrated using simple lotteries that utility functions are not globally concave. Rather, preference functions are concave over gains and convex (risk seeking) over losses. Third, people are significantly more averse to losses relative to the status quo than they are attracted by gains. That is, decision makers are loss averse. Lastly, empirical studies indicate that decision makers do not generally treat probabilities linearly. Instead, people tend to overweight small probabilities and underweight large probabilities. These findings lead to a new form of utility theory --- prospect theory.

Prospect theory was proposed first by Kahneman and Tversky (1979) (KT hereafter). One of the most widely used components of prospect theory is the value function which has three main characteristics:

- i. It is defined on deviations in wealth from a reference income level;

- ii. It is generally concave for gains and commonly convex for losses; and
- iii. It is steeper for losses than for gains;

The value function proposed by KT takes the form as represented in (2.1).

$$v(x) = \begin{cases} x^a & \text{for } x \geq 0 \\ -\lambda(-x)^a & \text{for } x < 0 \end{cases} \quad (2.1)$$

For $a < 1$, the decision maker with this value function is risk averse over gains and risk seeking over losses. Furthermore, individuals are more sensitive to losses than gains if the loss-aversion coefficient, λ , is greater than one (see Figure 2.1 for the value function in prospect theory). This function has found to be empirically robust, analytically convenient and the most common value function adopted in prospect theory analysis. A treatment of reference–dependent choice raises the question: what is the reference income? Although the reference state can be influenced by many factors like aspirations, expectations, norm and social comparisons, it is usually determined according to what defines gains and losses to decision makers. The decision makers in our crop-production setting are farmers who are either land owners or land renters. For both land owner and land renter, the cost of production is the land rent plus variable cost. The land rent would be the equilibrium price of land in a competitive land market which is approximately the expected annual returns to land. Farmers who rent land regard a revenue outcome as a gain if it is high enough to cover the rent and variable cost or as a loss if otherwise. Equivalently, farmers who own their land regard a profit outcome (returns over variable costs) as a gain if it is higher than the rent or as a loss if otherwise.

Another important element of prospect theory is the probability weighting function, π ,

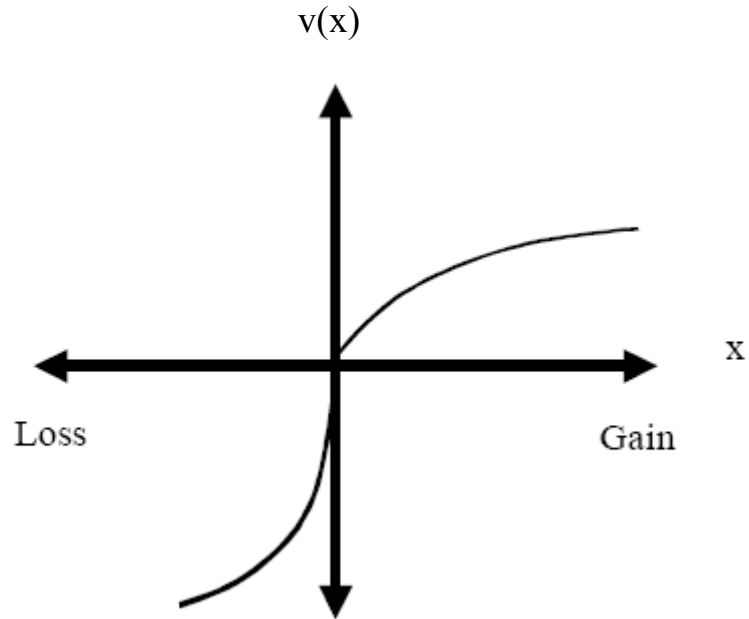


Figure 2.1. A hypothetical prospect theory value function
Source: Kahneman, Daniel, and Amos Tversky. 1979. "Prospect Theory:
an Analysis of Decision under Risk." *Econometrica* 47(2): 279.

which transforms state probabilities into decision weights and is of the following form in KT (1979):

$$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \quad (2.2)$$

The weighting function (2.2) is an increasing function of state probability p , with $\pi(0) = 0$ and $\pi(1) = 1$. That is, outcomes contingent on an impossible event are ignored. But this weighting scheme is just a monotonic transformation of outcome probabilities, which has two problems. First, sometimes it does not satisfy stochastic dominance, an assumption that many are reluctant to give up. Second, it is not readily extended to prospects with a large number of outcomes (KT 1992). Both problems can be easily solved by using the rank-dependent or cumulative functional form. Instead of transforming outcome probability

directly, KT (1992) transforms cumulative probabilities separately to gains and losses using weighting function (2.3).

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1+p)^\gamma)^{1/\gamma}}, \quad w^-(p) = \frac{p^\delta}{(p^\delta + (1+p)^\delta)^{1/\delta}} \quad (2.3)$$

Prelec (1998) proposes an alternative specification---the compound-invariant weighting function as represented by (2.4). While p denotes the state probability associated with a monetary outcome in (2.2), we use the same symbol p to denote the cumulative probability associated with a monetary outcome in (2.3) and (2.4).

$$w^+(p) = \exp(-\beta^+ (-\ln p)^\alpha), \quad w^-(p) = \exp(-\beta^- (-\ln p)^\alpha) \quad (2.4)$$

In (2.4), α represents probability sensitivity that accounts for the shape or the curvature of the weighting function. β^+ and β^- represent probability attractiveness that acts to shift the entire weighting function up and down relative to the expected utility argument. The shape of this two-parameter-form weighting function is nearly identical to that of KT's. But the axiomatic functional form of Prelec's has a theoretical advantage, that is, they order different classes of expected utility violations in the same way. Luce (2001) provides a simpler derivation of this weighting function based on reduction invariance, which turned out to be equivalent to the two parameter weighting function that Prelec derived. Al-Nowaihi and Dhimi (2005) gave an even simpler derivation based on power invariance, which is shown to be equivalent to the two parameter weighting functions that Prelec derived also.

Of all the weighting functions that have been proposed, that of Prelec's has the advantages that it is parsimonious, consistent with much of the empirical evidence, and has an

axiomatic foundation. Therefore, Prelec's two-parameter weighting functional form is chosen in this study.

Now, we have a Cumulative Prospect Theory (CPT) representation which involves three continuously increasing scaling functions: a value function $v(x)$, which gives the value of a monetary outcome, and two probability weighting functions $w^+(p)$ and $w^-(p)$, which transform cumulative probabilities into decision weights for gains and losses respectively.

2.4 Implementation and calibration of cumulative prospect theory

A prospect is a finite distribution over outcomes: $(x, p; y, q; \dots)$ assigning probability $p, q \dots$ to outcomes $x, y \dots$. The CPT representation of preferences is of the following form for a prospect with only two outcomes:

$$v(x, p; y, q) = \begin{cases} w^+(p+q)v(x) + w^+(q)(v(y) - v(x)), & 0 < x < y & (2.5a) \\ w^-(p+q)v(x) + w^-(q)(v(y) - v(x)), & y < x < 0 & (2.5b) \\ w^-(p)v(x) + w^+(q)v(y), & x < 0 < y & (2.6) \end{cases}$$

Equation (2.5) and (2.6) assume a "rank- and sign-dependent" framing of outcomes. If x and y have opposite sign, as in equation 2.6, then the prospect is called a mixed prospect and framed as a p -chance of losing value of x and q -chance of gaining value of y . If, however, both x and y are gains, or both are losses, as in equation 2.5a or 2.5b, then the prospect $(x, p; y, q)$ is called a pure positive prospect (or a pure negative prospect) and framed as a $p + q$ chance of gaining (or losing) at least the value of the middle outcome $v(x)$, and a q chance of gaining (or losing) an extra $v(y) - v(x)$. In CPT, the argument of the weighting function is not the probability of obtaining outcome x but the cumulated probability of obtaining an outcome at least as good as x , if x is positive, or at least as bad as

x, if x is negative. For the simulations in our model, the implementation of CPT is much more complicated because we have 5,000 outcomes.

We present a generalization of equation 2.5 and 2.6 to show the implementation of CPT with many outcomes in Monte Carlo analysis. First, price and yield data (5,000 draws for each variable) are generated for corn and soybeans respectively following the same procedures as discussed in the Methodology section that found that alfalfa hay does not enter into the solution. The maximum expected profit is the profit level where the farmer plants half of his land to corn and the other half to soybeans. Using the maximum expected profit as the reference income level, gains and losses can be obtained by subtracting reference income from simulated profits. To illustrate more clearly, suppose there are 2300 gains and 2700 losses. Because the weighting scheme is rank- and sign-dependent, we sort gains and losses in ascending order and denote the ordered outcomes as $L_{2700}, L_{2699}, \dots, L_2, L_1, G_1, G_2, \dots, G_{2299}, G_{2300}$. Recall that the weighting scheme transforms cumulative probabilities instead of state probabilities into decision weights. So, in order to know the decision weight for each monetary outcome, we need to know the corresponding cumulative probability. According to equation 2.5 and equation 2.6, the cumulative probability associated with $G_1 (L_1)$ is $2300/5000$ ($2700/5000$) and that with $G_2 (L_2)$ is $2299/5000$ ($2699/5000$). The cumulative probability associated with $G_{2300} (L_{2700})$ is $1/5000$ ($1/5000$). Given cumulative probabilities, we can derive decision weights for those monetary outcomes by plugging the cumulative probabilities into $w^+(p) = \exp(-\beta^+(-\ln p)^\alpha)$ and $w^-(p) = \exp(-\beta^-(-\ln p)^\alpha)$ separately. Similarly a value measure for each outcome is obtained by plugging gains or losses into the positive or negative part of the value function. With a value measure and a decision weight

assigned for each monetary outcome, the expected value of this mixed prospect can be obtained according to equation (2.7). Certainty equivalent returns (CER) can be derived according to the specification of the value function. If the expected value is greater than zero, we apply the positive part specification of the value function to get CER. Otherwise, we apply the negative part specification of the value function to get CER. The risk premium ratio is simply the ratio of CER to standard deviation of profit.

$$EV = w^- \left(\frac{2700}{5000} \right) v(L_1) + w^- \left(\frac{2699}{5000} \right) (v(L_2) - v(L_1)) + \dots + w^- \left(\frac{1}{5000} \right) (v(L_{2700}) - v(L_{2699})) \\ + w^+ \left(\frac{2300}{5000} \right) v(G_1) + w^+ \left(\frac{2299}{5000} \right) (v(G_2) - v(G_1)) + \dots + w^+ \left(\frac{1}{5000} \right) (v(G_{2300}) - v(G_{2299})) \quad (2.7)$$

After implementing CPT in our model, we now calibrate parameters to demonstrate why the CPT framework can capture decision makers' preferences over risk in a globally consistent way.

In CPT representation, we have a total of five parameters to calibrate. The value function has two parameters: the risk attitude parameter a and the loss aversion parameter λ . The probability weighting function has three parameters: the probability sensitivity parameter, α , which characterizes people's probability weighting behavior, and the probability attractiveness parameter β . These last two parameters shift the entire weighting function up or down relative to probabilities which are used in expected utility.

There clearly is an identification problem in trying to calibrate all five parameters when there is no structural relationship among them. The approach that we take is to use the existing literature as a guide. Wickham (2004) uses empirical experiments to test several hypotheses related to the five parameters and suggests:

- i. Given conventional risk behavior, the majority of subjects will be risk averse over gains; parametrically, $0 < a < 1$.
- ii. Given conventional probability weighting behavior, the majority of subjects will demonstrate over-weighting of low probabilities and under-weighting of moderate to high probabilities; parametrically, $0 < \alpha < 1$, and α is independent of outcome value.
- iii. It is not appropriate to assume β is equal to one.
- iv. The measured value of the risk attitude parameter a , and probability attractiveness parameter β^+ and β^- , are not independent of monetary outcome values.

Because the probability sensitivity parameter α is independent of monetary outcome value, it is useful to calibrate this parameter first. Tversky and Kahneman (1992) gave subjects many different pairs of choices in an effort to derive certainty equivalent returns for gambles. They estimated that the probability sensitiveness parameter α should be 0.61 for gains and 0.69 for losses. Camerer and Ho (1994) used data from nine studies and gave an estimate of $\alpha = 0.56$. Wu and Gonzalez's (1996) estimated $\alpha = 0.74$. They also fit the data to Prelec's weighting function and report 0.74 as an estimate of α . Prelec (1998) used his own experiment data yielding a value of 0.6 for α . Although the specifications of weighting function may be different in these studies, these estimates are for the same thing, the probability sensitiveness parameter, which characterizes people's probability weighting behavior. Together with these studies and Wickham's proposition 2, we use 0.65 as a reasonable value for α .

There is relatively little controversy about the loss aversion coefficient λ . We simply follow Tversky and Kahneman (1992) which suggests that λ is equal to 2.25. This number implies that the decision maker values the pain of a dollar loss 2.25 times as much as the value of a dollar gain.

Selection of values for α and λ simplify the choice of the remaining three parameters. Wickman's proposition 3 indicates that a , β^+ and β^- are not independent of monetary outcome values. Therefore, our approach is to select a value for a and then solve for the value of β^+ such that we obtain a desired level (say 25%) for the risk premium ratio to capture risk aversion preferences over gains which, as a whole, is regarded as a pure positive prospect. Similarly, we select a value for a and calculate a corresponding value for β^- by assuming a -25% risk premium ratio for risk-seeking preferences over losses which, as a whole, is regarded as a pure negative prospect. The risk premium ratio is simply the difference between expected gains (losses) and CER for gains (losses) divided by standard deviation of gains (losses), depending on whether expected value is positive or negative. This approach will give reasonable results so long as the 25% risk premium ratio is believed to be a reasonable characterization of people's preferences over risk.

We first calibrate parameters for the farmer with a small stake, say only one acre of land. Setting different values for a , a corresponding β^+ and β^- can be calibrated as shown in Table 2.6 (for an assumed risk premium ratio of 25%).

It is readily seen that there are infinitely many sets of parameter values such that a risk premium ratio of 25% can be obtained.

We now calibrate parameters for a farmer with a large stake, say, 1,000 acres of land. Setting different values for a , a corresponding β^+ and β^- can be calibrated as shown in Table 2.7. It turns out that the same parameter values are obtained for each given a as those previously obtained for one acre of land. This implies that if we measure the risk premium ratio of the farmer with 1,000 acres of land at parameter values associated with one acre of land, or vice versa, we will get the 25% risk premium ratio, which we believe to be reasonable ex ante.

Put another way, there are infinitely many sets of parameter values for which a reasonable risk premium ratio of 25% can be obtained. No matter which set of parameter values we choose, the calibrated value function and probability weighting function is a globally consistent representation which can characterize farmers' preferences over risky alternatives. Brandstatter and Kuhberger (BK) (1999) proposed a view of the probability

Table 2.6. Parameter calibration for farmers with one acre of land

a	β^+	β^-
0.32	0.8960	0.8422
0.52	1.0473	0.9758
0.60	1.1062	1.0283
0.70	1.1788	1.0933
0.80	1.2504	1.1577
0.88	1.3071	1.2087
0.95	1.3563	1.2531

Table 2.7. Parameter calibration for farmers with 1,000 acres of land

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0.52	1.0473	0.9758
0.60	1.1062	1.0283
0.70	1.1788	1.0933
0.80	1.2504	1.1577
0.88	1.3071	1.2087
0.95	1.3563	1.2531

weighting function as a composite of cognitive and emotional “biases” or “errors” in preferences and arrive at the conclusion that nonlinearities in the weighting function of probabilities may be the consequence of an intelligent compromise to cognitive and emotional “biases” or “errors”. Our calibration results are consistent with BK’s findings. The problem now is which specific set of parameter values we should choose for our decision and welfare analysis. It is known that individuals with preferences that follow cumulative prospect theory bet on unlikely gains and insure against unlikely losses. Neilson and Stowe (2002) point out that in order for individuals to bet on unlikely gains and insure against unlikely losses, the risk attitude parameter a must be high. When a is low, the convexity of the weighting function cannot overcome the concavity of the utility function. The estimates for a obtained by Camerer and Ho (1994) (0.32) and Wu and Gonzales (1996) (0.52) are too low. The Tversky and Kahneman (1992) estimate of $a = 0.88$ allows for some betting on unlikely gains and some insurance against unlikely losses. We checked the insurance and bet activity in each above study using the simulated data and found similar results. Because there are infinitely many sets of parameter values giving the 25% risk premium ratio, and when $a = 0.88$, our simulated results give similar insurance and bet activity as in KT’s study, so a set of parameter values where $a = 0.88$ is just picked as shown in Table 2.8.

2.5 Decision and welfare analysis

With cumulative prospect theory and calibrated parameter values at hand, it is straightforward to carry out the decision and welfare analysis. We first look at the farmer’s

Table 2.8. The set of parameter values in CPT

λ	a	α	β^+	β^-
2.25	0.88	0.65	1.3071	1.2087

willingness to pay (WTP) for government programs. We do this by first considering the impact on optimal acreage choice and the resulting WTP for each government program in turn, assuming that no other program is in place, and then calculating optimal acreage choice and WTP assuming that all programs are offered together. The reference income is the expected profit (returns over variable costs) when there is no government program. After that, we estimate the decision maker's willingness to accept (WTA) the loss of government programs, first singly and then jointly. The reference income in this case is the expected profit when farmers have access to all government programs. Comparing the results in WTP scenario and WTA scenario, we can obtain estimates of the endowment effects of government programs.

Using Monte Carlo simulation, acreages for corn (as a percentage of total land) and for soybeans are found that maximize farmer's expected profit and expected value. Thus the maximum expected profit, maximum expected value and CER can be obtained. Changes in CER and changes in expect profits can be easily derived by subtraction. Changes in CER are taken as the measure of program benefits while changes in expected profit are taken as the measure of program costs. The efficiency with which government redistributes income can be measured by the increase in producer welfare per dollar of government spending. Namely, the ratio of the change in CER over the change in expected profits. Table 2.9 and Table 2.10

present the results assuming that there are “harsh” yield penalties from planting soybeans after soybeans or corn after corn.

The no program case in the WTP scenario and the all programs case in the WTA scenario can be regarded as base cases. Any acreage distortion effects can be analyzed by comparing the results in each scenario with the corresponding base case. With a 50% acreage allocation for each crop in every case, we find no acreage distortion effects from government programs, given a 10% yield penalty for corn planted after corn and a 12.5% yield penalty for soybeans planted after soybeans. Therefore, government programs act as lump-sum transfers to farmers with regard to their acreage decisions.

As far as the efficiency of individual programs is concerned, we see in Table 2.9 that farmers are willing to pay a maximum of \$10.59, \$4.93, \$6.43, \$11.66 and \$51.43 for DPs, CCPs, LDPs, CI and all programs together respectively, while the cost of providing one unit of those programs is \$23.68, \$9.73, \$14.48, \$14.71 and \$62.61, respectively.

Therefore, DPs, CCPs, LDPs, CI and all programs together increases producer’s welfare by about 0.45, 0.51, 0.44, 0.79 and 0.82 dollars for each dollar of government cost, respectively. Similar magnitude of efficiency measures can be obtained for WTA results as shown in Table 2.10.

The endowment effects can be seen from the comparison of results of WTP scenario and WTA scenario since the high reference income point of WTA scenario is due to the endowment of all farm programs. Comparing changes of CER and efficiency measure of Table 2.9 and 2.10, we see that with harsh yield penalties for continuous cropping, producer values farm programs more when he has the programs than when he doesn’t. The WTA

Table 2.9. Willingness to pay results when there is a harsh yield penalty (\$/acre)

Harsh penalty/WTP	no program	DPs	CCPs	LDPs	INSURANCE	All programs
Profit maximizing corn acreage share	50%	50%	50%	50%	50%	50%
Value maximizing corn acreage share	50%	50%	50%	50%	50%	50%
Expect Value (EV)	-29.37	-13.92	-22.37	-20.19	-12.26	21.63
CER	-18.53	-7.93	-13.60	-12.10	-6.87	32.90
Changes in CER		10.59	4.93	6.43	11.66	51.43
Expect Profit (EP)	195.79	219.47	205.52	210.27	210.50	258.39
Changes in EP		23.68	9.73	14.48	14.71	62.61
Efficiency		0.45 ¹	0.51	0.44	0.79	0.82
Reference income	195.79	195.79	195.79	195.79	195.79	195.79

Table 2.10. Willingness to accept results when there is a harsh yield penalty (\$/acre)

Harsh penalty/WTA	all programs	no insurance	no LDPs	no CCPs	no DPs	No program
Profit maximizing corn acreage share	50%	50%	50%	50%	50%	50%
Value maximizing corn acreage share	50%	50%	50%	50%	50%	50%
Expect Value (EV)	-19.22	-39.56	-30.95	-26.92	-36.74	-74.71
CER	-11.45	-25.99	-19.67	-16.78	-23.90	-53.53
Changes in CER		-14.55	-8.22	-5.33	-12.45	-42.09
Expect Profit (EP)	258.39	243.68	243.91	248.66	234.71	195.79
Changes in EP		-14.71	-14.48	-9.73	-23.68	-62.60
Efficiency		0.99	0.57	0.55	0.53	0.67
Reference income	258.39	258.39	258.39	258.39	258.39	258.39

valuation of the individual programs on one acre basis is \$1.86, \$0.40, \$1.79 and \$2.89

higher than WTP valuation for DPs, CCPs, LDPs and CI, respectively. In

another word, it leads to more pain to farmers to be deprived of those government programs

when farmers originally have access to them than joy to farmers by providing farm programs

when they are originally not available to farmers. Considering all programs together,

farmers' WTP is greater than WTA because, compared with the original state of no program,

all programs together move farmers from a state of loss to a state of gain. The value function

¹ The efficiency measure is scale-independent. The analysis here is on a one-acre land basis. The same values for the efficiency measure were obtained when a 5,000 acre farm was examined.

is flatter in the gains part than that in the losses part, a unit of expected value increase is associated with a larger increase in CER (WTP) in gains part. Put in another way, loss aversion leads farmers willing to pay much more for this move.

We considered above the general harsh penalty case. But, a portion of Iowa farmers experience a lighter penalty or even no penalty due to different locations or new technology available. A welfare and decision analysis for the no penalty case is carried out also. Table 2.11 and Table 2.12 present the results assuming that there are “no” yield penalties from

Table 2.11. Willingness to pay results when there is no yield penalty (\$/acre)

No penalty/WTP	no program	DPs	CCPs	LDPs	insurance	all programs
Profit maximizing corn acreage share	100%	100%	100%	100%	100%	100%
Value maximizing corn acreage share	100%	100%	100%	100%	100%	100%
Expect Value (EV)	-46.75	-27.62	-35.67	-36.14	-21.69	28.68
CER	-31.43	-17.28	-23.11	-23.45	-13.13	45.32
Changes in CER		14.14	8.32	7.97	18.30	76.75
Expect Profit (EP)	222.25	245.93	232.09	233.61	236.02	280.91
Changes in EP		23.68	9.84	11.36	13.78	58.66
Efficiency		0.60	0.85	0.70	1.33	1.31
Reference income	222.25	222.25	222.25	222.25	222.25	222.25

Table 2.12. Willingness to accept results when there is no yield penalty (\$/acre)

No penalty/WTA	all programs	no insurance	no LDPs	no CCPs	no DPs	no program
Profit maximizing corn acreage share	100%	100%	100%	100%	100%	100%
Value maximizing corn acreage share	100%	100%	100%	100%	100%	100%
Expect Value (EV)	-20.84	-54.66	-39.26	-31.63	-41.77	-96.01
CER	-12.55	-37.53	-25.77	-20.16	-27.65	-71.19
Changes in CER		-24.99	-13.22	-7.61	-15.10	-58.65
Expect Profit (EP)	280.91	267.14	269.55	271.07	257.23	222.25
Changes in EP		-13.78	-11.36	-9.84	-23.68	-58.66
Efficiency		1.81	1.16	0.77	0.64	1.00
Reference income	280.91	280.91	280.91	280.91	280.91	280.91

planting soybeans after soybeans or corn after corn.

The no program case in the WTP scenario and the all programs case in the WTA scenario can be regarded as base cases. Any acreage distortion effects can be analyzed by comparing the results in each scenario with the corresponding base case. The acreage decisions indicate that when there is no yield penalty, the farmer will choose to plant 100% of his land to corn because corn has an expected profit higher than that of soybeans. With a 100% acreage allocation for corn in every case, we find no acreage distortion effects from government programs when there is no-yield penalty for continuous cropping. Therefore, government programs act as lump-sum transfers to farmers with regard to their acreage decisions.

As far as efficiency of these programs is concerned, we see in Table 2.11 that farmers are willing to pay \$14.14, \$8.32, \$7.97, \$18.30 and \$76.75 for DPs, CCPs, LDPs, CI and all programs as a whole respectively, while the cost of providing one unit of those programs is \$23.68, \$9.84, \$11.36, \$13.78 and \$58.66 respectively. Therefore, DPs, CCPs, LDPs, CI and all programs as a whole increases producer's welfare by about 0.60, 0.85, 0.70, 1.33 and 1.31 dollars for each dollar of government cost, respectively. Similar magnitude of efficiency measures can be obtained for WTA results as shown in Table 2.12.

The endowment effects can be seen from the comparison of results of WTP scenario and WTA scenario since the high reference income point of WTA scenario is due to the endowment of all farm programs. Comparing changes of CER and efficiency measure of Tables 2.11 and 2.12, we see that with no yield penalty for continuous cropping, producer values farm programs more when he has a higher endowment except for CCP program. The WTA valuation of the individual programs on one acre basis is \$0.96, \$5.25 and \$6.69 higher

than WTP valuation for DPs, LDPs and CI, respectively. In another word, it leads to more pain to farmers to be deprived of those government programs when farmers originally have access to them than joy to farmers by providing farm programs when they are originally not available to farmers. Considering all programs together, farmers' WTP is greater than WTA because, compared with the original state of no program, all programs together move farmers from a state of loss to a state of gain. Loss aversion leads farmers willing to pay much more for this move.

One problem that arose with these results is the low willingness to pay for DPs. Most observers would assume that the willingness to pay for DPs should be close to one because DPs are a lump-sum payment. But, in our analysis, we found efficiency measures much lower than one. After breaking down the efficiency analysis, we find that there are three reasons which account for the low efficiency measure for DPs and for all other individual programs.

First, as KT (1992) mentioned, for both pure positive and pure negative prospects, the decision weights add to one, For mixed prospects, however, the sum can be either smaller or greater than one, because the decision weights for gains and for losses are defined by separate functions. In our mixed prospect, the sum of decision weights turns out to be 0.74. This implies that when we calculate the expected value of the mixed prospect, we eliminate 26% of the expected value at the very beginning because the sum of weights equal to 0.74 instead of one. Farmer's willingness to pay thus the efficiency measure is lower accordingly. The efficiency measure for DPs is 0.62 after increasing all decision weights by a factor of $1/0.74$ so that the sum of weights is equal to one. Second, without any government program, the farmer is originally in a state of loss on average because of the skewed yield distribution.

The payments from any single individual government program is small which only move the farmer from the state of loss to another state of loss where loss aversion is a key factor for farmers to evaluate the welfare of government programs. Loss aversion coefficient shrinks the WTP for programs by a factor of $\left(\frac{1}{2.25}\right)^{\left(\frac{1}{0.88}\right)}$. The efficiency measure is lower

accordingly given the fixed cost of programs. The third reason for low efficiency is that direct payments (and the other program payments as well) are made not only when there is a loss but also when there is a gain. Expected value is essentially the weighted average of all values. Loss aversion leads to more value associated with a dollar increase in the loss part than that in the gains part. In addition, the calibrated weighting scheme where β^+ is greater than β^- results in a higher decision weight for a loss than for a gain of the same order.

Therefore, the change in expected value is lower when many of payments are made in the gains part relative to the change in expected value when all the payments are made in the loss part. Put in another way, a dollar payment is of higher efficiency when there is a loss than when there is a gain. An investigation showed that the proportion of payments made in the gains part for DPs, CCPs and LDPs are 43.2%, 23.5% and 24.7%, respectively. For CI, only 9.7% of the payments are made in the gains part, which account for the highest efficiency of CI among those government programs. To verify that it is because of payments made in the gains part that leads to the low efficiency measure of these programs, we performed a WTP analysis for revenue insurance which is designed to provide protection to farmers against the risk of low profit. Payments are triggered when the profit from crops is lower than the expected profit. There is no payment made if vice versa. The efficiency of the revenue insurance varies from 1.10 to 3.25 at different levels of premium subsidy rate which takes

value in the range from 0.1 to 1.0. For this revenue insurance, every dollar payment is made in the loss situation. Thus a much higher efficiency measure is obtained.

2.6 Summary of results

In sum, we find that for our representative Iowa farmer, agronomy dominates. There is no policy distortion to farmers' acreage decisions. Therefore, government programs act as lump-sum transfers to farmers with regard to their acreage decisions.

The efficiency measures of individual government programs are low because of three reasons. First, for our mixed prospect, the sum of decision weights turns out to be 0.74. This implies that when we calculate the expected value of the mixed prospect, we eliminate 26% of the expected value at the very beginning because the sum of weights equal to 0.74 instead of one. Farmer's willingness to pay thus the efficiency measure is lower accordingly. Second, the payment from each of the government programs only moves the farmer from a state of loss to another state of loss where loss aversion is a key factor for farmers to evaluate the welfare of a government programs. Loss aversion coefficient shrinks the WTP for programs by a factor of 1/2.25. The efficiency measure is lower accordingly given the fixed cost of programs. And third, the payments from these government programs are made not only when there is a loss but also when there is a gain. Expected value is essentially the weighted average of all values. Loss aversion leads to more value associated with a dollar increase in the loss part than that in the gains part. In addition, the calibrated weighting scheme where β^+ is greater than β^- results in a higher decision weight for a loss than for a gain of the same order. Therefore, the change in expected value is lower when many of payments are made in the gains part relative to the change in expected value when all the payments are made in the

loss part. Put in another way, a dollar payment is of higher efficiency when made in the loss part than in the gains part. Crop insurance has the highest efficiency among these individual programs because the proportion of payments made in the gains part are much smaller compared with that of DPs, CCPs and LDPs.

The endowment effects can be seen from the comparison of results of WTP scenario and WTA scenario since the high reference income point of WTA scenario is due to the endowment of all farm programs. Comparing changes of CER and efficiency measure of Table 2.9 and 2.10, Table 2.11 and 2.12, except for CCP program in no-penalty case, farmers value government programs more when he has a higher endowment. In another word, it leads to more pain to farmers to be deprived of those government programs when farmers originally have access to them than joy to farmers by providing farm programs when they are originally not available to farmers. Considering all programs together, however, farmers' WTP is greater than WTA because, compared with the original state of no program, all programs together move farmers from a state of loss to a state of gain. Loss aversion leads farmers willing to pay much more for this move.

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CHAPTER 3. A METHOD FOR MEASURING POOLABILITY OF SPATIALLY CORRELATED RISK

3.1 Introduction

There is a widespread belief that the risks of crop losses cannot be effectively pooled by private insurance companies, which is one justification given for public support of the crop insurance industry. Numerous studies have examined this and other possible reasons for market failure. Among the first explanations is market failure attributed to asymmetric information problems, particularly adverse selection and moral hazard problems (Skees and Reed 1986; Chambers 1989; Nelson and Loehman 1987; Goodwin and Smith 1995). Moral hazard occurs when the insured changes his behavior after purchasing the insurance so that the probability of receiving an indemnity increases. Adverse selection occurs when farmers have more information about the risk of loss than the insurer does. Adverse selection describes the situation where farmers who realize that their expected indemnity will exceed premium are more likely to buy insurance than those who don't. Harwood et al. (1999) think that the reasons, on the demand side, why private crop insurance markets have not developed is due to the high cost of crop insurance relative to other alternative risk management strategies to mitigate the risk that they face. These include futures and options markets, contracting, crop and livestock diversification, non-farm income, federal price and income support programs, and federal disaster assistance payments.

More recent explanations impute market failure to systemic nature of risk inherent into crop insurance (Mirander and Glauber 1997; Duncan and Meyers 2000; Skees and Barnett 1999; and Mason et al 2003). Systemic risk in agriculture stems primarily from the impact of

geographically extensive unfavorable weather events, such as droughts, floods or extreme temperatures, which induce significant correlation among individual yield losses. One of the necessary conditions for a risk to be poolable is that there are a large number of roughly homogeneous, independent exposure units so that the law of large numbers can provide an accurate prediction of average future losses. Insurance markets are better suited for sharing uncorrelated risks such as automobile accident and property damage due to fire and wind. In the classic sense, crop yields are not insurable because losses are correlated which violates the independence condition of insurance Skees and Barnett (1999). In their 1997 study, Miranda and Glauber define a measure of systemic risk as the ratio of the actual coefficient of variation of indemnity paid to the hypothetical one when insured losses were independent. A ratio of one indicates no systemic risk. Systemic risk increases as the ratio increases above one. They use a stochastic simulation model to generate U.S. corn, soybeans and wheat yield draws with existing correlation imposed and compute coefficient of variation of total indemnities paid by the 10 largest crop insurers under a 35% deductible crop insurance program in the U.S. The systemic risk ratios indicate that U.S. crop insurers face portfolio risk anywhere from 22 to 49 times larger than if indemnities were independent. They conclude that the high level of systemic risk undermines a crop insurer's ability to diversify risk across space, and prevents it from performing the essential function: the pooling of risk across individuals. Motivated by this study, Mason et al. (2003) break down the total risk absorbed by the U.S. crop insurance industry into poolable and systemic components. Various hedging strategies using speculative markets are examined for their potential to hedge the systemic risk accepted by the government. The results indicate that the risk reduction achievable by hedging is appreciable. Duncan and Myers (2000) develop an

insurance model to investigate the role of catastrophic risk in contributing to inadequate or incomplete crop insurance coverage. A long-run insurance market equilibrium is defined assuming risk averse farmers and insurance firms because of incomplete opportunities for diversifying or capitalizing the catastrophic risk. Results indicate that high levels of catastrophic risk can reduce coverage levels, increase premium and, if high enough, lead to complete breakdown of the market. This occurs because catastrophic risk causes insurance firms to act as if they are risk averse and average risk can not be reduced simply by expanding their portfolio of crop insurance contracts.

Wang and Zhang (2003) doubt the belief that a private market for crop insurance is doomed to fail because of the systemic risk existing in crop yield. In their paper, they take a spatial statistics approach to examine the effectiveness of risk pooling for crop insurance under correlation. They argue that a critical ingredient for risk pooling is that the portfolio variance of mean losses decreases as exposure units increases. So, one way to measure the effectiveness of risk pooling is to see how small the variance of mean losses is compared with the average variance of each individual loss. Their empirical study shows that the yields for the three crops (corn, soybeans and wheat) present zero or negative correlation when two counties are far apart, which complies with a weaker condition than independence, finite-range positive dependence. Hence, they impute the failure to effectively pool crop loss risk is the insurers' failure to sufficiently diversify risk across space.

The Wang and Zhang (2003) results imply that the existence of systemic risk does not necessarily justify Federal intervention in U.S. crop insurance markets. But it could be that the costs of achieving sufficient spatial diversification so that aggregate risks are acceptable

to either an insurance company alone or an insurance company working with a private re-insurance company, are so high as to be prohibitive.

One practical cost in achieving sufficient poolability is the actual measurement of aggregate risk in any insurance company's book of business. The approaches of Miranda and Glauber and Wang and Zhang require fairly sophisticated quantitative tools of analysis. Presumably, insurance companies would need to apply these methods to existing and prospective books of business to determine the degree of aggregate risk for each. The objective of this paper is the development of an alternative approach for determining the aggregate risk of a book of business. Development of such a method should lower the cost of assessing aggregate risk of any give book of business.

The method is developed using hail loss cost data of corn for each township of Iowa in years 1985 to 2000. Loss cost is defined as the ratio of losses paid out to insured liability. It is the proportion of the premium rate that is applicable solely to loss, without provision for company expenses or profits. The existence of a private hail insurance market in the United States indicates that hail-caused yield loss risk can be made poolable. The essential difference between crop insurance and hail insurance lies in the level of risk dependence across space (Skees and Barnett). The higher the level of risk dependence across space, the lower the risk poolability. Given a book of business, hail insurance providers could assess its degree of poolability using a stochastic simulation model of insurance indemnities to compute the variability of total indemnities paid. However, this procedure is complicated and time-consuming because it requires development of hail simulation models, estimation of the degree of spatial covariance and trend. Indemnities need to be simulated and proper distributions of involved variables need to be investigated. In addition, the procedure needs

to be repeated in order to evaluate risk poolability for every book of business. This paper develops a method to measure risk poolability of any specific book of business quickly by simply knowing a few key statistics of the given book of business.

The development proceeds as follows. First, a spatial variogram is estimated and a theoretical model of loss cost resulting from hail damage in Iowa is fit to the empirical data. Thus, we explicitly model hail loss using a spatial statistical approach. Next, hail losses are simulated for a wide range of books of business using Monte Carlo simulation. And finally, a regression model is estimated that captures the essence of the Monte Carlo simulation. This model can then be used to quickly estimate the degree of poolability of any given book of business.

3.2 Methodology

3.2.1 Spatial concepts

The degree of spatial dependence of loss cost is the key factor determining risk poolability of a book of business in hail insurance. What is spatial dependence? An intuitive answer is that things are (positively) spatially dependent if quantities that are located close together are more similar than quantities that are located farther apart. If loss costs of customers are independent, then the risk for the book of business is insurable. The higher the spatial dependence of loss cost, the less able are insurance companies to pool their risks across customers.

To measure spatial dependence, we use the spatial variogram which is defined as

follows. For locations s_i and s_j and associated random variables $Z(s_i)$ and $Z(s_j)$, the variogram is

$$\gamma(s_i - s_j) \equiv \frac{1}{2} \text{var}\{Z(s_i) - Z(s_j)\} \quad \text{all } s_i, s_j \in D$$

where D is a collection of points of R^d at which some type of “event” has occurred.

To calculate a sample variogram, suppose that $E\{Z(s)\} = \mu; \forall s \in D$, then we would have

$$\gamma(s_i - s_j) = \frac{1}{2} E[Z(s_i) - Z(s_j)]^2 \quad (3.1)$$

(3.1) suggests using the average of squared differences in data values to represent $\gamma(\cdot)$. For a

given displacement h , if we have observations from a set of pairs of

locations $N(h) \equiv \{(s_i, s_j) : s_i - s_j = h; i, j = 1, \dots, n\}$, this is in fact what we would do, and a

sample variogram for this displacement would be

$$\gamma(h) = \frac{1}{2|N(h)|} \sum_{N(h)} \{Z(s_i) - Z(s_j)\}^2 \quad h \in R^d,$$

where $|N(h)|$ is the number of pairs in the set $N(h)$. Without repeated observations at the same displacement, we will not have sets $N(h)$. In fact, typically we will not have more than a few pairs of observations with the same displacement. To deal with this problem, we use the idea of tolerance region. Suppose, in a given set of data, there are m distinct displacement values. Choose k of these values $h(l), l = 1, \dots, k$, and define tolerance regions as $T(h(l)), l = 1, \dots, k$, then the set of paired locations with distance within the corresponding tolerance region can be defined as

$$N(h(l)) \equiv \{(s_i, s_j) : s_i - s_j \in T(h(l))\}, l = 1, \dots, k.$$

We will consider two of the most common variogram models in this analysis. Both models make the assumption that displacement between two locations $(s_i - s_j)$ enters the variogram only in terms of distance $d_{ij} = |(s_i - s_j)|$. The first is the exponential variogram model

$$\gamma(h | \theta) = \begin{cases} 0 & h = 0 \\ a + b(1 - \exp(-|h|/c)) & h \neq 0 \end{cases}$$

Where $\theta = (a, b, c)$, with $a \geq 0, b \geq 0$ and $c \geq 0$. The second is the spherical variogram model

$$\gamma(h | \theta) = \begin{cases} 0 & h = 0 \\ a + b(1.5 \cdot (|h|/c) - 0.5 \cdot (|h|/c)^3) & 0 < h \leq c \\ a + b & h \geq c \end{cases}$$

where $\theta = (a, b, c)$, with $a \geq 0, b \geq 0$ and $c \geq 0$. Both variogram models are characterized by the same set of parameters as follows. The range c is the distance at which the variogram reaches a maximum (if it does). The interpretation of the range is that it is the distance beyond which $Z(s+h)$ and $Z(s)$ are no longer correlated. The sill $a+b$ is the value of the variogram when evaluated at any distance greater than or equal to the range. The sill is usually a constant number. The nugget a presents micro-scale variance. Clearly, $\gamma(0) = 0$ if $|h|=0$. But it may not be true that $\gamma(h/\theta) \rightarrow 0$ as $|h| \rightarrow 0$. If $\gamma(h/\theta) \rightarrow a$ as $|h| \rightarrow 0$, then a is called the nugget. The nugget is how much of the variation of the response variable cannot be explained spatially. Figure 3.1 illustrates how the shape of the variogram changes as the distance increases for exponential and spherical models, where the x-axis denotes distance and the y-axis denotes the value of variogram.

3.2.2 Variogram estimation

Loss cost data is available only at the township level of disaggregation. To give a better understanding of the size of a township, we include a map of townships in Jasper County in Iowa as shown in Figure 3.2. There are 36 sections (640 acres per section) in a township with one square mile for each section. Thus a perfectly square township is 36 square miles in area. This means that the distance from the center of a township to the center of a neighboring township is about 10 kilometers. There are a total of 1,626 townships in Iowa. Hail loss cost data of corn for each township in Iowa from 1985 to 2000 is obtained from NCIS (National Crop Insurance Services). For each township, we use the X and Y coordinates of the area centroid to indicate the location of the corresponding township. The X and Y coordinates are expressed in terms of Euclidian distance from a location in northeast corner of state Nebraska.

It may be the case that hail loss costs exhibit spatial trend, increasing in one direction or another. Figure 3.3 maps the average hail loss cost by township over all the years



Figure 3.1. Exponential variogram model (left) and spherical variogram model (right)

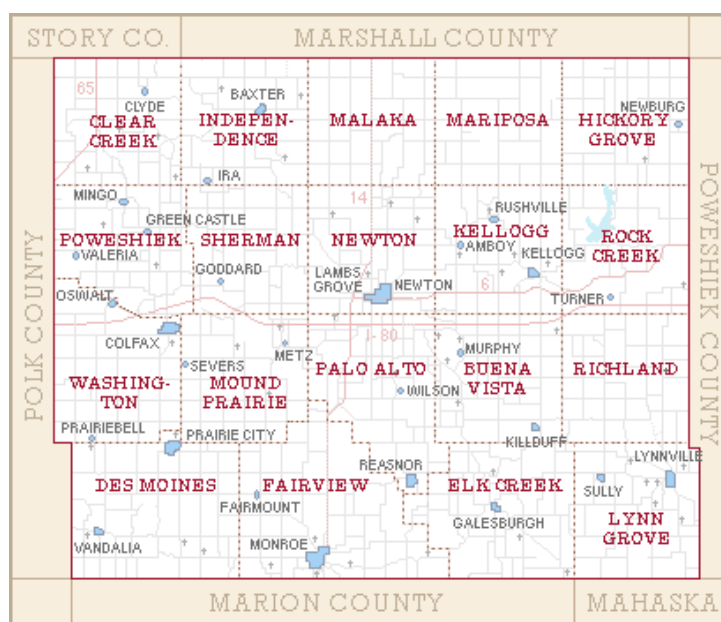


Figure 3.2. Townships in Jasper County, Iowa

investigated. It is clearly showed that the average loss cost is higher in Northwest, West Central and South Central sections of Iowa, and it gets lower and lower as we move to the East Central and Southeast sections of the state.

To measure spatial correlation of loss cost, we need to detrend the loss cost data and carry out analysis based on the residuals. Generally, the median polish method is used to remove the spatial trend (Cressie, 1993). However, we can not use this method to detrend the data since more than 50 percent of the loss cost observations are zero each year. Thus the median is also zero. Instead, we fit a Tobit model to estimate the trend.

The no program case in the WTP scenario and the all programs case in the WTA scenario can be regarded as base cases. Any acreage distortion effects can be analyzed by comparing the results in each scenario with the corresponding base case. The acreage decisions indicate that when there is no yield penalty, the farmer will choose to plant 100%

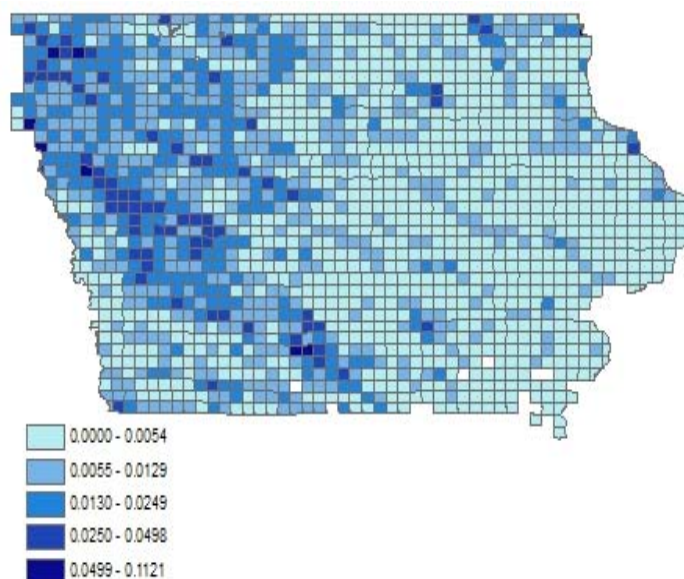


Figure 3.3. Average hail loss cost of townships in Iowa from 1985 to 2000

of his land to corn because corn has an expected profit higher than that of soybeans. With a 100% acreage allocation for corn in every case, we find no acreage

The no program case in the WTP scenario and the all programs case in the WTA scenario can be regarded as base cases. Any acreage distortion effects can be analyzed by comparing the results in each scenario with the corresponding base case. The acreage decisions indicate that when there is no yield penalty, the farmer will choose to plant 100% of his land to corn because corn has an expected profit higher than that of soybeans. With a 100% acreage allocation for corn in every case, we find no acreage

The reason for choosing a Tobit model is that the loss cost takes either positive values or zeros which is actually a censored dependent variable with all the negative values being reported as zero. Namely, for each individual year, loss cost is regressed on X and Y coordinates according to the following linear form

$$LC = \beta_0 + \beta_1 \cdot X + \beta_2 \cdot Y + \varepsilon \quad (3.2)$$

In (3.2), LC is the loss cost of hail damage which takes value of either zero or positive numbers, X and Y are the X and Y coordinates of the area centroid of each township. The estimation is carried out through a toolbox, econometrics, associated with the Matlab software. After fitting the model, we performed a likelihood ratio test with the null hypothesis of $\beta_1 = \beta_2 = 0$. We conclude that there is no spatial trend in loss cost if we fail to reject the null hypothesis. Otherwise we conclude that there is spatial trend for the specific year. The year-by-year regression and likelihood ratio test indicate that there is spatial trend in loss cost for each year from 1985 to 2000. Therefore, we detrend the data by subtracting the predicted loss costs which are the fitted values of linear regression (3.2) from the original loss costs to get the residuals.

Now for each year, we have detrended loss cost for each township in Iowa. Together with location of each township, variogram estimation can be carried out through the use of “geoR” package associated with R software which provides programs to calculate spatial variograms. We first estimate spatial variogram of loss cost assuming isotropy which means that the displacement between two locations enters the variogram only in terms of distance, but not in terms of direction. We divide all possible paired distances into 10 bins with 0-10 kilometers as the first bin, 10-20 kilometers as the second bin, and so on. For any given township, the number of townships within 10 kilometers is about eight. Of course, the number of paired townships with distance within 10 kilometers will be much larger if considering all possible paired townships. Instead of computing variogram of pairs of points which have exactly the same distance, we calculate variogram for pairs of points which have a distance within a range, for instance, 0-10 kilometers, 10-20 kilometers, and so on. Each bin can be regarded as a tolerance region of the mid-point distance of the bin. A user of the

program is asked to provide parameter values for the minimum distance, the maximum distance and the number of bins considered. The minimum distance and the maximum distance define the range of distances at which variogram will be estimated. The maximum distance is set at 100 kilometers by trial and error which is much larger than the estimated range. When the paired distance is this big, there is surely no spatial dependence. The minimum distance and the number of bins can be decided according to the tradeoff between big bins and small bins. Big bins imply a lot of squared differences to average across so that the estimated variogram associated with the mid-point distance of the bin has smaller variance. Small bins imply that the estimated variograms at distances in the bin are good approximation of variogram associated with the mid-point distance of the bin. We then estimate variogram assuming anisotropy which means that the displacement between two locations enters the variogram not only in terms of distance, but also in terms of direction. Accordingly, in addition to a division of 10 bins, we also divide distances according to directions. For a specific point, we consider totally four different directions: horizontal direction (0-degree direction), vertical direction (90-degree direction), north-east to south-west direction (45-degree direction) and north-west to south-east direction (135-degree direction). In each direction, isotropy is assumed. We apply the concept of tolerance region here which is 45-degree around each of the four directions in addition to division of bins. For instance, for the north-east to south-west direction, we consider all the points which are located in the fan-shape area from 22.5 degree to 67.5 degree and in the fan-shape area from 202.5 degree to 247.5 degree. In this region, we calculate the variogram for pairs of points which have distances within a bin, for instance, 0-10 kilometers, 10-20 kilometers, and so on.

3.2.3 Test for isotropy

For each year, we can get a vector of variograms for the isotropy case and four vectors of variogram for the anisotropy case, with one vector for each direction. The variogram vectors have a dimension of 1-by-10 since we set the number of bins equal to 10 in the program. Stacking the 16 estimated variograms for each bin of the ten bins for both isotropy case and the four anisotropy cases, we can perform a hypothesis test with the null hypothesis that the spatial variogram of loss cost is isotropic. We apply ANOVA to two regressions. One is an unrestricted regression. The other is a restricted regression with the restriction that the corresponding model parameters for each of the four directions are equal. We apply the spherical model to do the hypotheses test first because this model is most commonly used. The exponential model will be used to do the hypothesis test later. If the p-value of the F statistic from the ANOVA table is greater than 0.05, we conclude, at a 5% significance level, that there are no significant difference between corresponding parameter values for the four directions and thus the spatial variogram is isotropic. Otherwise, we conclude there is significant difference between corresponding parameter values for the four directions and the spatial variogram is anisotropic. (3.3) gives the unrestricted regression in terms of exponential model in each direction.

$$Y = D_0 \cdot (a_0 + b_0(1 - \exp(-|h|/c_0))) + D_{45} \cdot (a_{45} + b_{45}(1 - \exp(-|h|/c_{45}))) + D_{90} \cdot (a_{90} + b_{90}(1 - \exp(-|h|/c_{90}))) + D_{135} \cdot (a_{135} + b_{135}(1 - \exp(-|h|/c_{135}))) \quad (3.3)$$

where Y is the response variable, here the stacked directional variogram in our problem. $|h|$, the explanatory variable, is the pair distances. Recall that we divide all possible distances into 10 bins. For each bin, we use the mid value of the bin as distance $|h|$. For example, for bin

with a range from 0 to 10 kilometers, $|h|=5$, and for bin with a range from 10 to 20 kilometers, $|h|=15$, etc. D_0, D_{45}, D_{90} and D_{135} are dummy variables indicating which direction is under consideration. I use subscript 0, 45, 90 and 135 to represent the four directions: 0-degree direction, 45-degree direction, 90-degree direction and 135-degree direction. a_0, a_{45}, a_{90} and a_{135} are nugget parameter of the spherical model for corresponding directions. c_0, c_{45}, c_{90} and c_{135} are range parameter of the model for corresponding directions. $a_0 + b_0, a_{45} + b_{45}, a_{90} + b_{90}$ and $a_{135} + b_{135}$ are sill parameter of the model for corresponding directions. If $a_0 = a_{45} = a_{90} = a_{135}$, $b_0 = b_{45} = b_{90} = b_{135}$ and $c_0 = c_{45} = c_{90} = c_{135}$, we get the restricted regression as (3.4).

$$Y = a + b(1 - \exp(-|h|/c)) \quad (3.4)$$

where Y is the response variable, the stacked isotropic variogram. $|h|$ is still the explanatory variable which is the distance between pair of points. $a, a + b$ and c are corresponding model parameters of nugget, sill and range.

The p-value of the F statistic in the ANOVA table is 0.276 for spherical model, which is larger than 0.05. Therefore, we fail to reject the null hypothesis at 5% significance level and conclude that there is no statistical evidence supporting anisotropy. The p-value of the F statistic from the ANOVA analysis is 0.281 for exponential model. We fail to reject the null hypothesis at 5% significance level and conclude again that the variogram of loss cost is isotropic. Thus we assume that loss cost is isotropic.

3.2.4 Variogram models

Given isotropy, we choose between exponential model and spherical model to fit the detrended loss cost data. The estimation results are shown in Table 3.1 and Table 3.2. Parameters estimated for both models are a , b and c which are the nugget, difference of sill and nugget, and range of the models. The second and the third column give the parameter estimates and corresponding standard error of the estimates. The fourth and fifth column give the t statistics and corresponding p-values. The last column shows the significance levels of the estimates. It is clear that the spherical model fits the data better than exponential model does since the p-value of each parameter estimate is smaller in spherical model than in exponential model. The significance level of those estimates in the spherical model is close to or less than 5%. The estimated nugget in the spherical model is 4.24. The estimated sill is 7.93, which is also the maximum variogram. The nugget indicates that about 48% of the variation of the response variable can be explained spatially. The estimated range is 41.02 kilometers which implies that, at this distance, the spatial variogram reaches the maximum and loss costs are no longer correlated when the corresponding locations are greater than 41 kilometers, which is a distance of about four townships. Figure 3.4 shows

Table 3.1. Estimation results of exponential model

Parameter	Estimate	Std. Error	t value	Pr(> t)
a	3.56	2.49	1.43	0.15
b	4.47	2.34	1.91	0.06
c	15.15	14.25	1.06	0.29

Table 3.2. Estimation results of spherical model

Parameter	Estimate	Std. Error	t value	Pr(> t)
a	4.24	1.62	2.62	0.01
b	3.69	1.65	2.34	0.027
c	41.02	22.66	1.81	0.07

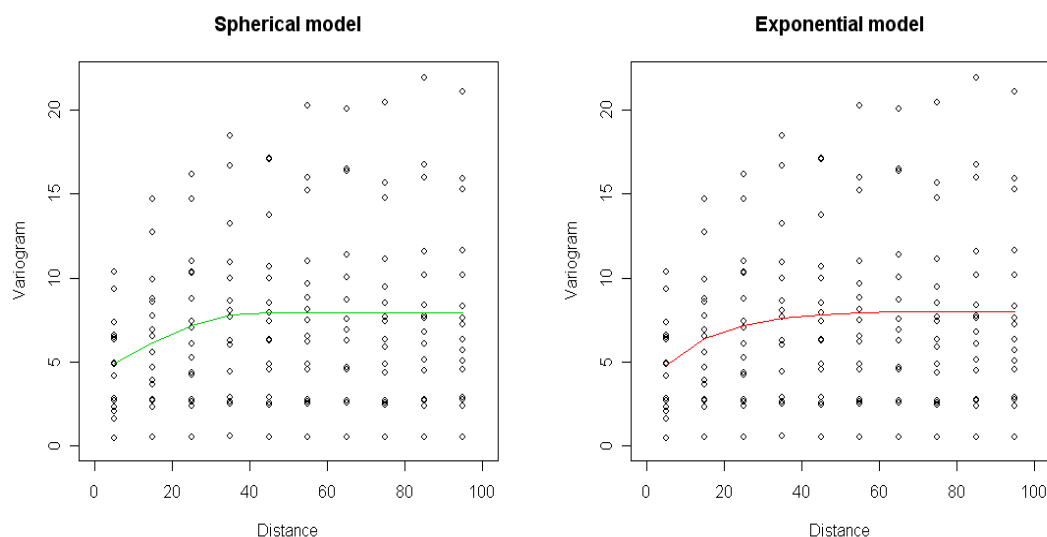


Figure 3.4. Spatial variogram model fit

the fit of the two models. It is clear that the estimated range is larger in spherical model than that in exponential model.

3.3 Monte Carlo simulation of a wide range of books of business

The primary objective of this study is to develop a method for easily estimating the degree of risk poolability given an insurance portfolio. For a given book of business, the hypothetical crop insurer faces a portfolio risk which can be measured by the coefficient of variation of total indemnities paid:

$$CV_I = \frac{\sqrt{V(\sum_i I_i)}}{E(\sum_i I_i)}$$

where I_i is the random indemnity payment to customer i . If the risks are independent across insured customers, the coefficient of variation of total indemnities is zero if enough customers are insured. Otherwise, the coefficient of variation is greater than zero. For hail

insurance, the higher the spatial correlation of lost cost, the higher the coefficient of variation of indemnities.

The objective of this study is to develop a method to quickly assess the poolability of a book of business. Accomplishing this requires the simulation of a series of books of business with a range of coefficients of variation of indemnities of hail insurance for different books of business in Iowa. In order to do the simulation, we need to understand first what the most important factors of a book of business are which affect its risk poolability. After investigation, we find out that the number of customers, the location concentration and acreage concentration of those customers play significant roles. The number of customers is simply the total number of customers in a given book of business. Because we assume that there is only one customer in each township, the number of customers is also the number of townships in the book of business. Location concentration measures how geographically disperse customers are for a given book of business. A book of business can be quite concentrated in a few townships or it can consist of customers that are widely spread over the entire state. Acreage concentration measures how disperse the size of customers are in terms of acreage insured for a given book of business. The combination of geographic dispersion along with a lack of acreage concentration creates a more diversified book of business. It is a very different situation when every customer has the same number of insured acres compared to when small number of customers has a large volume of insured acres in a book of business.

Thus we have many combinations of variability to simulate: variability in the number of customers, variability in location concentration and variability in acreage concentration.

The one thing fixed in the simulation is the total number of acres insured which is fixed at

1,000,000. The purpose of this simulation is to create a response variable which measures portfolio risks and a design matrix as well which contains the levels of explanatory variables measuring diversity of a book of business. An analogy is the rating method for Revenue Assurance. The premium rate for a given farm is a function of APH yield, APH rate, coverage levels and price volatility. The goal of this study is to relate portfolio risk to diversification measures of the portfolio based on Monte Carlo simulation of a large number of different books of business.

For a book of business with N customers, I vary location concentration by randomly sampling N customers from different subsets of the 1,590 townships. Different subsets are obtained by changing the ranges of X coordinate and Y coordinate simultaneously. The state as a whole is also a subset when the lower limits of X , Y coordinates are set as the minimum values of the coordinates and the upper limits of X , Y coordinates are set as the maximum values of the coordinates. The standard beta distribution with parameters p and q is chosen to simulate insured acreage for each customer in a book of business. To facilitate a mean-preserving spread in variability, we set $p=q$ and vary the values of p and q to get different acreage concentration. The beta distribution is ideal for this task because of its flexibility. A low variance density can be defined by setting high values of p and q . For example, $p=q=50$ represent a nearly uniform spatial distribution. We can choose a fat-tailed, unimodal density ($p=q=2$) to represent medium spatial variability and a U-shaped density ($p=q=0.5$) to represent a high degree of concentration. These three density functions are graphed in Figure 3.5.

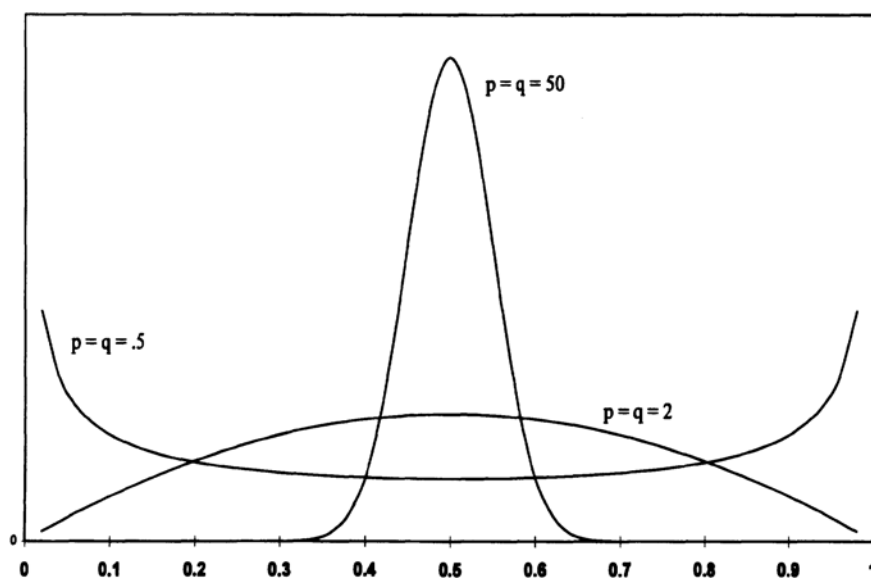


Figure 3.5. Three beta density functions measuring spatial variability

There are many different indexes to measure concentration. We are particularly interested in two of them. One is the out-of-range proportion index, the other is the Herfindahl index.

From the spatial models we know that when the distance between any two customers is greater than the estimated range, the hail damage losses are essentially independent no matter where the two customers are located. Thus the variance in total indemnities paid will be small. Inspired by this fact, we come up with the first measure, out-of-range proportion index, which is the weighted proportion of paired distances exceeding the estimated range. We will call this measure proportion index for simplicity. Specifically, if the distance between customer i and customer j is greater than the range, count it as one, otherwise, count it as zero. Weight the count by the insured acreage of customer i multiplied by the insured acreage of customer j . Summing over those weighted

counts, dividing by the weighted counts when all pair distances are greater than the range, we can get the proportion index. Mathematically,

$$\text{proportion_index} = \frac{\sum_{i=1}^n \sum_{j>i}^n I_{ij} a_i a_j}{\sum_{i=1}^n \sum_{j>i}^n a_i a_j}$$

where I_{ij} is the indicator variable which takes value of one if distance between customer i and customer j is greater than the range or zero otherwise; a_i and a_j are the insured acreage of customer i and customer j , respectively; n is the number of customers in a book of business. Proportion index takes values in a range from zero to one. For a given book of business, if all the customers are farther apart from each other than the estimated range, the measure takes the value of one. If all the customers are close together and all pair distances are smaller than the range, then the measure takes value of zero. Since the estimated range is only 41 kilometers and we assume that there is only one customer in each township, it is very unlikely to get a value as small as zero except when the number of customers in the book of business is really small and they are located in an area with the maximum pair distance smaller than 41 kilometers. It is obvious that proportion index depends on the number of customers and location concentration of a given book of business.

The Herfindahl index is a simple, yet sophisticated way of measuring acreage concentration. The Herfindahl index is obtained by squaring the written share of insurance of each customer, and then summing those squares.

$$\text{Herfindahl_Index} = \sum_{i=1}^n \left(\frac{a_i}{A}\right)^2$$

where a_i is the insured acreage of customer i and A is the total insured acreage of the given book of business. Thus $\frac{a_i}{A}$ is the written share of customer i and n is the number of customers for the book of business. For example, consider a hypothetical book of business with three equal-sized customers. Each of them has same acreage insured, thus each has a share of $1/3$. The Herfindahl index is computed as follows:

$$\text{Herfindahl_index} = (1/3)^2 + (1/3)^2 + (1/3)^2 = 1/3$$

Now, we are ready to carry out Monte Carlo simulations. Given historical lost cost for each township from year 1985 to 2000, taking average across years gives the value of average loss cost for each township, which is a good starting point for measuring the actuarially fair insurance rate for the township. There may be a big difference in average loss cost among townships that are close to each other, which pose a practical problem for insurance in real life. The reason is, for townships close to each other, the insurance rate tends to be similar. To solve this problem, a quadratic model relating average loss cost with X and Y coordinates is fitted to smooth average loss cost across space. The predicted loss cost is then taken as the insurance rate for corresponding townships. Figure 3.6 maps the insurance rate for each township in Iowa.

The historical probability that hail damage occurs in a specific township is obtained by taking the ratio of the count of years when damage occurred to the count of years when a loss cost record is available. This historical probability is called the threshold probability in this study. For some townships in some years, there is no record at all, we disregard them since we do not know whether there is damage or not. Townships which have

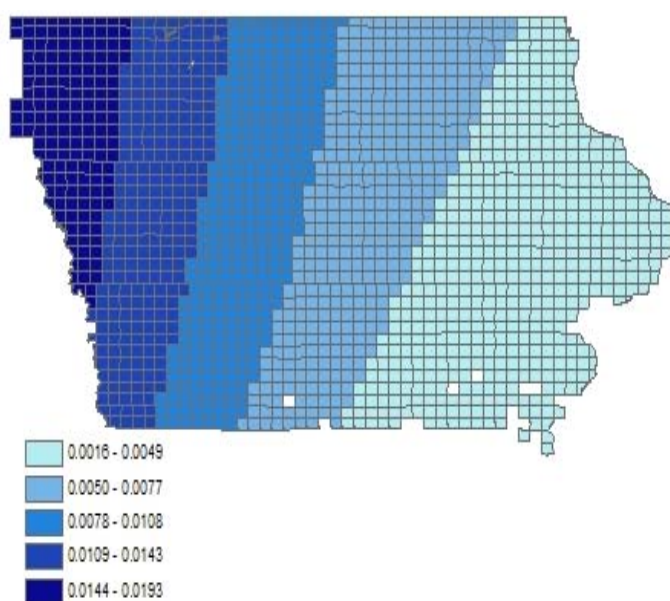


Figure 3.6. Predicted insurance rates of townships in Iowa

never had a hail-damage from 1985 to 2000 are excluded in this analysis. Therefore, a total of 1590 townships are the subjects of interest.

Conditional on a positive loss cost, loss cost is exponentially distributed (Benktander, 1977). Namely, the density function is of the form as represented by (3.5).

$$p(x) = \beta \cdot \exp(-\beta \cdot x) \quad (x \geq 0) \quad (3.5)$$

where $\beta > 0$ is the scale parameter of the distribution and the mean of the loss costs. Figure 3.7 shows the sample distribution of loss costs conditional on positive loss cost for a sample of adjacent thirty townships. With the predicted insurance rate and threshold probability for each township, we can easily solve for parameter beta by simply taking the ratio of insurance rate and threshold probability of the corresponding townships. For a specific township, conditional on positive loss cost, this calibrated value is the mean value of loss costs of the township.

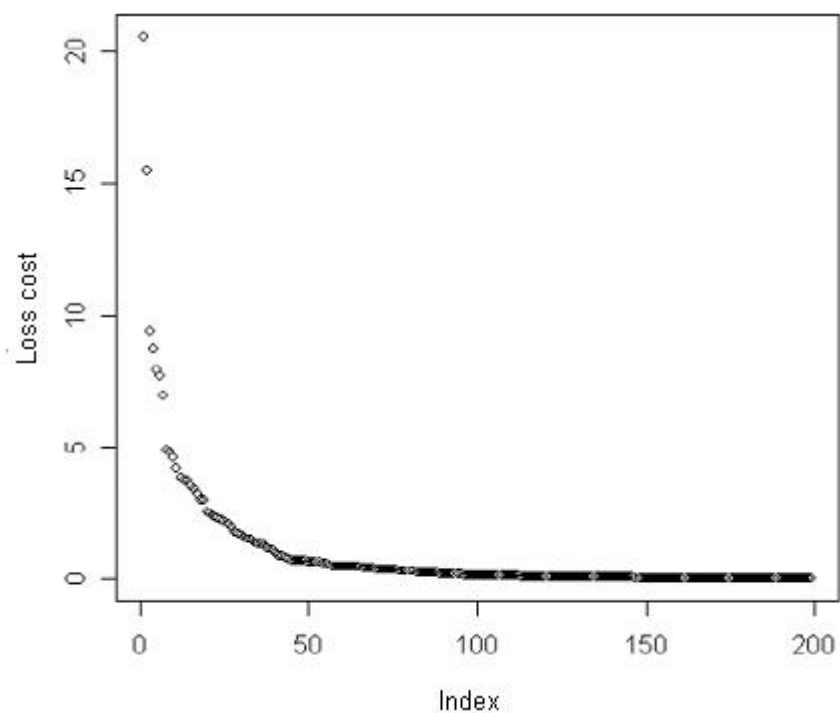


Figure 3.7. Sample distribution of loss costs conditional on positive loss cost

We assume that there is only one customer in each township. If there is more than one customer in a township, we can combine their acreage and regard them as one customer. Therefore, the maximum number of customers available in a book of business is 1590. With this assumption, the spatial correlation of hail damage for different townships (customers) can be appropriately imposed.

Given a book of business, pair-wise distance of the customers can be calculated by equation (3.6).

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (3.6)$$

where d_{ij} is the distance of township i and township j . (x_i, y_i) , (x_j, y_j) are the coordinate values of township i and township j , respectively. Pair-wise correlation of hail-damage loss

costs assuming spherical variogram model is a function of the corresponding pair-wise distance as shown in (3.7).

$$correlation_{ij} = \begin{cases} 1 - 1.5 \cdot d_{ij} / c + 0.5 \cdot (d_{ij} / c)^3 & d_{ij} \leq c \\ 0 & d_{ij} > c \end{cases} \quad (3.7)$$

where $correlation_{ij}$ is the spatial correlation of hail damage loss cost of township i and township j ; c is the estimated range beyond which there is no spatial correlation any more.

For a given book of business with N customers in N townships, we simulate loss cost for 500 years for each customer. To do this, first N columns of independent standard normal deviates are drawn. Each column has 500 rows corresponding to the 500 years. Cholesky decomposition is applied to the correlation matrix. Correlation is imposed upon the N columns of independent standard normal deviates to get N columns of correlated standard normal deviates. Then, we calculate the cumulative probability for each deviate column by column which has a range from zero to one. Recall that conditional on positive loss cost, loss cost is exponentially distributed with parameter beta. So, for a specific township (customer), we pick out those cumulative probabilities which are smaller than the threshold probability of the township and transform them to exponential deviates with expected value equal to the corresponding parameter beta of that township. For those cumulative probabilities which are greater than the threshold probability, we set corresponding loss cost as zero. In this way, for a given township, we have a simulation where there is a threshold probability that hail damage occurs and a probability of one minus threshold probability that no hail damage occurs at all. The same procedure is applied to other townships. The simulated indemnities of a book of business can be calculated by the equation: Indemnity=Liability \times simulated loss cost \times acre shares of the customers in the book of business. The result is a vector with

dimension of 500-by-1. Each row corresponds to an indemnity for a single simulated year. In our simulation, we set liability as 100 for simplicity. Finally, we are able to calculate the coefficient of variation of indemnities for the book of business. Given simulated indemnities, the coefficient variation of indemnities is equal to the standard deviation of indemnities divided by mean value of indemnities.

The proportion index and the Herfindahl index of a book of business can be computed accordingly. Recall that the standard beta distribution is chosen to simulate insured acreage for each customer and the total number of acreage insured is set as 1,000,000. For a book of business with N customers, N acreage deviates can be drawn according to the standard beta distribution. Acreage concentration can be varied by setting different values to the distribution parameters. Dividing each beta deviate by the sum of all N deviates, we can get acreage share of each customer. Squaring those shares and summing over the squares gives the Herfindahl index of the book of business. Consequently, insured acreage of each customer can be obtained by multiplying corresponding acreage share with the total number of acreage insured. Together with information of pair distance of customers, proportion index for the book of business can be calculated according to the corresponding formula.

3.4 Analytical results

For a given book of business, the proportion index which measures the weighted proportion of pair distances exceeding the range, the Herfindahl index which measures the acreage concentration and the coefficient of variation of indemnities can be computed. This given book of business is counted as one case. 10,000 different books of business are simulated in this study. Now, we are ready to fit a model which represents the relationship

between the coefficient of variation of indemnities and the portfolio diversification measures. The coefficient of variation of indemnities of a book of business is the response variable (Y). The portfolio diversification variables are the explanatory variables, here the proportion index (X_1) and the Herfindahl index (X_2). The models and the adjusted R^2 from the corresponding regressions are reported in Tables 3.3, 3.4 and 3.5.

First, the proportion index is included as the only explanatory variable in the model. We fit a linear model, a quadratic model and a power model in turn. The corresponding adjusted R^2 from the regression varies from 0.13 to 0.21. This implies that the proportion index can only explain about 20% of the variation in the coefficient of variation of indemnities. We then use the Herfindahl index as the only explanatory variable and fit linear model, quadratic model and power model in turn. The adjusted R^2 from each of the three models are high, varying from 0.85 to 0.98. Compared with the proportion index, Herfindahl index accounts for much more of the variation in the coefficient of variation of indemnities. Last, we include both proportion index and Herfindahl index in the regression and fit a linear model, quadratic model and Cobb-Douglas model one by one. The adjusted R^2 from each of the three models are quite similar to that of the fitted models when Herfindahl index is the only explanatory variable.

From the adjusted R^2 in the above three tables, it is obvious that Cobb-Douglas model or log-transformed model with both Herfindahl index and proportion index in the regression has the best fit of the nine possible models. The adjusted R^2 is 0.9855, which indicates that about 98% of the variation in coefficient of variation of indemnity can be explained by proportion index and Herfindahl index both in logarithm. The estimated parameters of the

Table 3.3. Models including only the proportion index

Model	Adjusted R^2
$Y = \beta_0 + \beta_1 \cdot X_1$	0.2034
$Y = \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_1^2$	0.2093
$Y = \alpha \cdot X_1^\beta$ or $\log Y = \log \alpha + \beta \cdot \log X_1$	0.1377

Table 3.4. Models including only the Herfindahl index

Model	Adjusted R^2
$Y = \beta_0 + \beta_1 \cdot X_2$	0.8538
$Y = \beta_0 + \beta_1 \cdot X_2 + \beta_2 \cdot X_2^2$	0.9185
$Y = \alpha \cdot X_2^\beta$ or $\log Y = \log \alpha + \beta \cdot \log X_2$	0.9832

Table 3.5. Models with both explanatory variables

Model	Adjusted R^2
$Y = \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_2$	0.8854
$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1^2 + \beta_4 X_2^2 + \beta_5 X_1 X_2$	0.9351
$Y = \alpha \cdot X_1^{\beta_1} X_2^{\beta_2}$ or $\log Y = \log \alpha + \beta_1 \log X_1 + \beta_2 \log X_2$	0.9855

Cobb-Douglas model are shown in Table 3.6. All the three regression coefficients are significant at 1% significance level since all corresponding p- values are smaller than 0.01. In addition, the correlation between $\log X_1$ and $\log X_2$ is -0.33. Therefore, multicollinearity is not a big issue here.

Table 3.6. Parameter estimation of the Cobb-Douglas model

Variable	DF	Parameter Est.	Std Err.	t	Pr> t
Intercept	1	0.5735	0.0128	74.7	<.0001
LogX1	1	-0.1997	0.0145	-49.6	<.0001
LogX2	1	0.4513	0.0006	751.5	<.0001

We fit models to several other sets of simulations and get results which are very similar. Therefore, we choose the Cobb-Douglas model. Namely, the coefficient of variation of indemnities has the relationship with proportion index and Herfindahl index for a book of business as represented in (3.8):

$$Y = 1.7745 \cdot X_1^{-0.1997} \cdot X_2^{0.4513} \quad (3.8)$$

where Y is coefficient of variation of indemnities, X_1 is proportion index and X_2 is Herfindahl index. The coefficient of 1.7745 is obtained by taking exponential of the estimated value of the intercept.

One issue with these results is that when the Herfindahl index as the only explanatory variable is included in the regression, the adjusted R^2 is as high as 0.983. This means the Herfindahl index by itself can explain 98% of the variation in coefficient of variation of indemnities of a book of business. One may wonder why we bother to have more than one explanatory variable. It seems there is very little left for the proportion index to explain after adjusting for the effects of Herfindahl index on both proportion index and response variable. We use proportion index to indicate the location concentration of the book of business. We include this measure in the model because we believe this measure is a very important diversification measure for portfolios of risk. The reason why proportion index seems to have little explanatory power in our model is that, in hail insurance, the range beyond which spatial correlation does not exist is quite small relative to the size of Iowa. It is only 41 kilometers in Iowa, which is a distance of about four townships. Most customers of an ordinary book of business are far apart from each other than the range. When the pair distances of customers are greater than the range, the locations of the customers do not matter

any more. Thus a high value of proportion index is an indication that location concentration is not a big issue for the book of business. When the range is really small, the proportion index takes values which clump to percentages as high as one hundred. In our simulation of different books of business, the proportion index rarely takes small values as shown in Figure 3.8 which is the very situation that proportion index really matters. For example, there is a book of business with only ten customers. Nine of them are close together with a maximum distance less than the range. The other one is far away from the rest of the nine. Suppose that hail damage occurs to one of the nine customers, chances are hail damage occurs to all the other neighboring customers. The risk of this book of business mainly lies in the risk of the

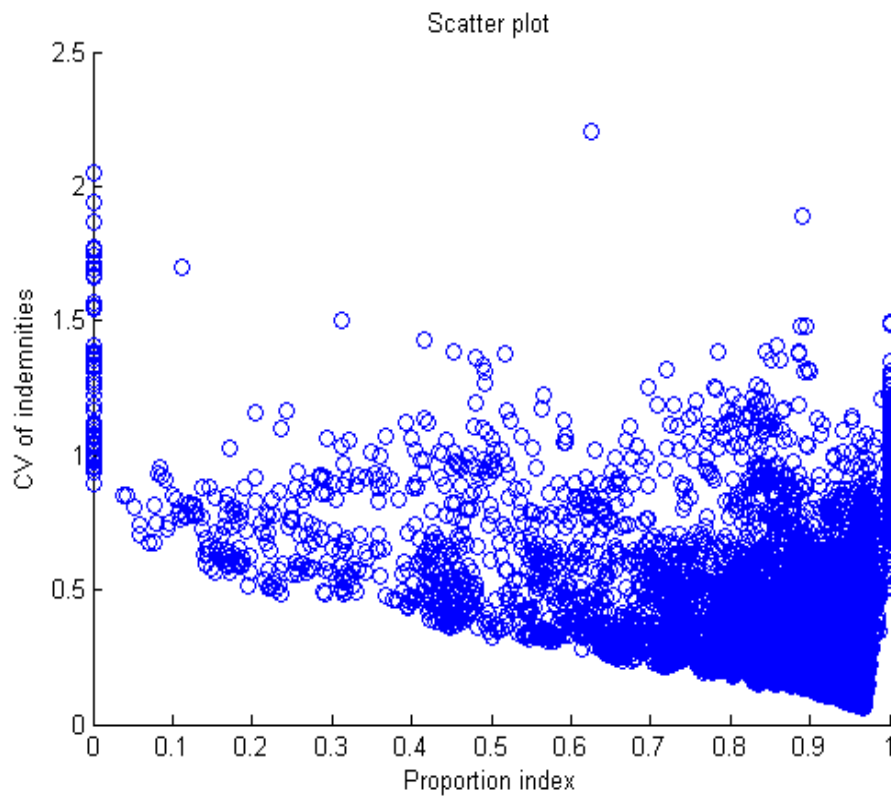


Figure 3.8. Scatter plot of CV of indemnities vs. proportion index

nine close-located customers. The location concentration of this book of business is very high and proportion index very low, which is the situation that proportion index matters. To confirm this idea, I checked the simulation to see what happened when the simulated proportion index is small. I pick out the smallest 1,000 proportion index and corresponding coefficient of variation of indemnities. Regressing coefficient of variation of indemnities onto proportion index, both in logarithm form, I get an adjusted R^2 as high as 0.316. This value implies that proportion index in logarithm can explain 31.6% of the variation of response variable in logarithm. This value is much higher than 0.1377 which is what I get from the same regression using all the simulated proportion index and corresponding coefficient of variation. This shows that proportion index measure does matter when the location concentration of a book of business is high.

In addition to hail insurance, there are other lines of insurances of which portfolio risks are determined by the risk dependence across space, particularly, crop insurance. For crop loss risk, the range is much larger than that for hail-caused loss risk. A study by Wang and Zhang (2003) shows that the crop yields have a clear pattern of finite range positive dependence and the distance for the positive dependence is at most 570 miles. There are many more cases where the proportion index takes smaller value than in the simulation of hail insurance portfolios. Since the goal of this paper is to illustrate a method which can quickly tell the degree of risk poolability of a book of business with spatial correlated losses involved, proportion index is needed in the model as a very important measure of portfolio diversification.

There is a reason why the Herfindahl index as the only explanatory variable can explain about 98% of the variation of response variable. The Herfindahl index takes on values which

are concentrated close to zero because if each customer in a portfolio has similar insured acreage, then the Herfindahl index value turns to be low. In our simulation, more than 90% of the cases have Herfindahl index values less than 0.2 as shown in Figure 3.9. These simulated cases play a decisive role in determining the relationship between coefficient of variation and Herfindahl index. It turns out that the good fit occurs when the Herfindahl index takes a value in this range. Therefore, it is reasonable to use both proportion index and Herfindahl index to measure the diversification of a book of business.

Provided with the fitted model, how can we measure the risk poolability of a book of business in hail insurance? Given a book of business, the number of customers, the location

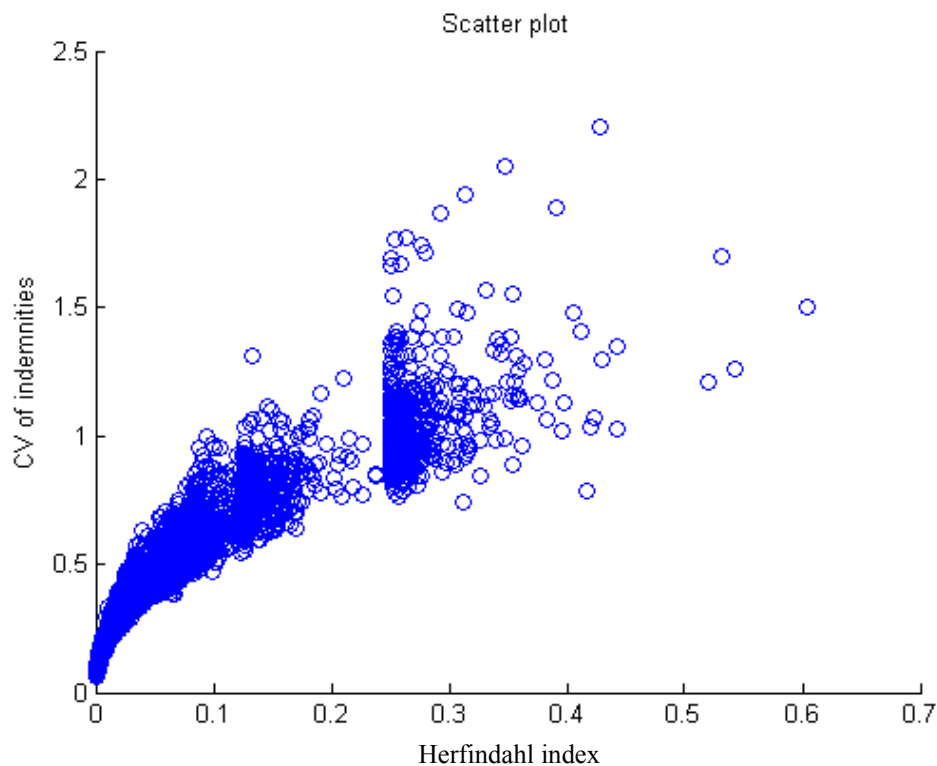


Figure 3.9. Scatter plot of CV of indemnities vs. Herfindahl index

of customers and size of each customer are known. The proportion index and Herfindahl index can be easily calculated according to the formulas. Plugging the values of the proportion index and Herfindahl index of a book of business's into the model, we can get, on average, a coefficient of variation of indemnities paid for the book of business. Miranda and Glauber (1997) report that the coefficient of variation of total indemnities paid is 15% on average for conventional crop-hail insurance in the United States. Our maximum simulated coefficient of variation of indemnities is higher than 15%. We choose 20% as the critical value and conclude that if the predicted value is greater than 20%, the book of business is not insurable, otherwise, it is. In our simulation, the coefficient of variation of indemnities paid for different books of business vary from 6% to 220%. Books of business with smaller number of customers who locate closely and have very different insured acreage tend to have higher coefficient of variation of indemnities. While, books of business with large number of customers who locate far away from each other and have similar insured acreage are more likely to have smaller percentage variability. For example, a book of business with 30 customers who are closely located in the east-north corner of the state of Iowa with high degree of concentration of incurred acreage has a 53.2% coefficient of variation of indemnities. Another book of business with 1,000 customers from widespread area of Iowa and have uniformly distributed insured acreage has a 9% coefficient of variation of indemnities. While the first case is not insurable, the second case is insurable.

3.5 Conclusions

Much work has been done on crop insurance in recent years. Only a couple of studies put effort on how to measure the risk poolability. No attempt has been given to measure risk

poolability of a given book of business in an easy and quick fashion. People can surely simulate losses and compute indemnities based on those draws for a given book of business. But this procedure is complicated and time-consuming. This study aims at providing a way to measure the risk poolability of a given book of business in an easy and quick fashion.

Diversification plays a decisive role in determining the risk poolability of a given book of business. It turns out that the number of customers, the locations and insured acreage of each customer are important factors which determine the diversification of a book of business. Two measures are constructed based on those factors to measure diversification. One is proportion index which measures the weighted proportion of pair distances exceeding the estimated spatial range. The other is Herfindahl index which measures the concentration of acreage across customers. After investigation, I find that a Cobb-Douglas model capture the simulation procedure quite well. About 98% of variability of the risk poolability can be explained by the proportion index and Herfindahl index of the book of business.

Although it is derived based on hail loss cost data from the state of Iowa, the model is directly applicable to a larger geographic area, assuming similar pattern of spatial correlation, in the sense that both proportion index and Herfindahl index are unitless. For crop insurance or other lines of insurance in which risk is spatially related, we can still apply this method to derive a new model to predict risk poolability of a given book of business. Of course, investigation of the spatial correlation of the corresponding risk is needed in constructing the new models.

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CHAPTER 4. REDUCING BASIS RISK IN WEATHER INDEX INSURANCE CONTRACTS

4.1 Introduction

Many argue that private crop insurance markets can not develop because of the systemic nature of risks inherent in crop insurance and because of asymmetric information problems (Skees and Reed, 1986; Nelson and Loehman, 1987; Chambers, 1989; Goodwin and Smith, 1995; Mirander and Glauber, 1997; Skees and Barnett, 1999; Duncan and Meyer, 2000; Mason et al., 2003). Systemic risk in agriculture stems primarily from the impact of spatially correlated unfavorable weather events which result in high correlation among farm-level yields, thereby defeating insurer efforts to pool risks across farms. Moral hazard and adverse selection induce high transaction costs to insurers who ultimately must pass these expenses onto insurance purchasers by loading premium rates. Despite these problems, government support crop insurance because it provides ex ante risk protection, rather than ex post forms of disaster assistance. For example, the U.S. government highly subsidizes premium to encourage participation in the crop insurance program. However, as a mechanism for providing subsidies, government premium subsidies are often inefficient and associated with high social cost (Hennessy, Babcock and Hayes, 1997; Hennessy, 1998; Skees, Hazell, and Miranda, 1999; Vedenov and Barnett, 2004).

Concerns over the systemic nature of crop risks and costs of insuring farm-level crop yield have prompted area-based insurance products such as the Group Risk Plan (GRP) and Group Risk Income Protection (GRIP) offered under the Federal Crop Insurance Program in the United States. Although these products overcome information asymmetry problems, the

insurance they provide at the county level results in high basis risk. In addition, they still suffer from a lack of poolability of insured risks.

In recent years interest has increased in weather-based index contracts as alternative risk-management instrument in agricultural production. The underlying weather index is usually a weather variable or a function of multiple weather variables accumulated over a period of time. There are many advantages of weather index contracts over the traditional individual-yield and area-yield crop insurance. First, individual who use an index contract should be unable to influence the outcome that determines payment from the contract. Monitoring needs are reduced, which lowers transaction costs. Second, the indemnity structure is not directly tied to actual crop yield. This saves lost adjustment costs and eliminates the possibility of moral hazard. Adverse selection is minimized or eliminated because premium calculation is based on objective weather events which are independent of participation of producers in the program. And third, aggregating production risks across space may reduce the idiosyncratic risk in the aggregate portfolio, the insurer can then hedge the systemic weather risk using weather derivatives.

The major concern associated with index contract is basis risk. In this sense, weather index insurance is similar to area-based insurance products. There are two layers of weather basis risk. The first layer, or the spatial basis risk, refers to the fact the weather index value defined at a weather station may not be the same as the realized weather index value at a specified location. The second layer, or the technological basis risk, refers to the fact that the underlying weather index is an imperfect hedge against risk exposure even if the underlying index and exposure being hedged correspond to the same location. As a result, producers

may not receive an indemnity even if he/she suffers a production loss, or alternatively, may receive an indemnity even though no loss has occurred.

Many studies have been conducted about the potential application of weather index insurance in agricultural production. Most focus on the economic, pricing and institutional requirement of weather-related insurance products for agriculture (Skees et al., 2001; Mahul, 2001; Martin, Barnett and Coble, 2001; Miranda and Vedenov, 2001; Turvey, 2001; Richards et al., 2004; Campbell and Diebold, 2005; Turvey, 2005). Only a handful of studies address the efficiency of weather derivatives as risk management instruments for agricultural production (Vedenov and Barnett, 2004; Woodard and Garcia, 2006; Deng et al., 2007). However, these studies either investigate hedging with non-local contracts or analyze farms where a weather station is available. Non-local contracts are inevitably plagued with the spatial basis risk problem. Efficiency analysis at a chosen farm where a weather station is available is of limited usage because the results can not be directly applied to other farms where weather station is not available.

Basis risk has been cited as a primary concern for the implementation of weather hedges (Turvey, 2001; Brockett et al., 2004; Turvey, 2006; Deng et al., 2007). However, most of these studies put emphasis on solving the second layer of basis risk, leaving spatial basis risk as still an open question. As Richards et al. (2004) mentioned, economic research can do little to remedy the spatial basis risk problem. Paulson (2006) was the first to explore how to reduce spatial basis risk employing sophisticated spatial interpolation techniques. Paulson (2006) analyzed the spatial basis risk for a rainfall insurance policy designed to protect pastureland owner of Iowa against drought risk. The interpolation technique employed in his study, Bayesian kriging prediction, is one of the stochastic prediction

methods in geo-statistics (Cressie, 1993). There are two main approaches to address spatial dependence problems. One is the standard, well-developed geo-statistical approach. The other is the less-developed Markov random field (MRF) approach. The geo-statistical analysis is appropriate for data defined on both continuous and discrete fields as long as the data analyzed are generated from same stochastic process. The MRF approach explicitly models spatial dependence of data defined on lattice. Both geo-statistical approach and MRF approach have strengths and weaknesses in terms of operational and data-analytic aspects. Cressie et al. (1999) developed spatial prediction models for particulate matter from both geo-statistical approach and MRF approach. Their results indicate that the MRF approach to spatial prediction has promise in the sense that MRF model yields cross-validation prediction with smaller prediction mean squared error than that of geo-statistical approach. The more automated nature of estimation with the MRF model makes it easier to model multiple years of data.

In this study, a temperature insurance contract is designed to provide protection to corn growers in Iowa. The contract is essentially an exotic call option on the temperature index, cooling degree days, accumulated during the summer season (ACDD). The number of cooling degree days on a single day can be calculated as the difference of the daily average temperature and 65 degrees Fahrenheit. The insurance policy is rated using Monte Carlo simulation assuming that ACDD at a given location follows normal distribution. ACDD is interpolated for locations where recorded ACDD data is not available through MRF prediction. Both the geo-statistical model and the MRF model in addition to a naive multiple linear regression model, which assumes no underlying spatial correlation of ACDD, are fitted to the historical data. The results indicate that MRF is preferred in the sense that it has a

lower cross-validation prediction mean squared error and that it provides a straightforward extension to multiple years of data in model estimation.

Although the emphasis of this analysis is on addressing spatial basis risk, technological basis risk is also addressed through the use of indemnity factor which can be obtained through linear regression relating corn yield losses to the temperature index rises. Potential performance of the proposed insurance policy is investigated through historical analysis from 1980 to 2005. Given the high-level systemic nature of temperature, the proposed insurance policy is better supplied by large insurance companies or government agencies in the sense that a large portion of the aggregated portfolio's total risk is left in the form of systemic weather risk after aggregating yield risks across space. The systemic weather risk could be securitized via the Chicago Mercantile Exchange and over-the-counter security markets.

4.2 Relationship between ACDD and corn yields

It is well accepted that high temperatures during the growing season can significantly hinder corn development. Agronomic experiments indicate that cooling degree days are more relevant to crop yields than outright temperature measurements (Schlenker et al., 2006). In this study, empirical evidence indicates that cooling degree days accumulated over the months June, July and August has a high correlation with corn yields in Iowa. For simplicity, we use ACDD to denote accumulated cooling degree days during the summer months. Data of corn yields in each of the nine Crop Reporting Districts (CRDs) in Iowa from 1980 to 2006 is available from USDA-NASS. To account for the temporal and technology component, a simple linear trend is fit to corn yields for each CRD. The detrended 2006 equivalent yields are then calculated by equation (4.1).

$$Y_t^{\text{det}} = Y_t \frac{Y_{2006}^{\text{tr}}}{Y_t^{\text{tr}}} \quad (4.1)$$

Scatter plots of corn yields versus ACDD for every CRDs are investigated. The nine scatter plots have similar pattern with regard to the relationship between corn yields and ACDD across CRDs. Therefore, we only report the integrated scatter plot for the whole state as in Figure 4.1.

The numbers in the figure are the last two digits of corresponding years of data involved. Figure 4.1 indicates that year 1993 corresponds to an outlier when the ACDD level is pretty normal but corn yields are the lowest in the historical time period investigated. It can be explained by the extreme moisture pressure due to the wet weather in year 1993.

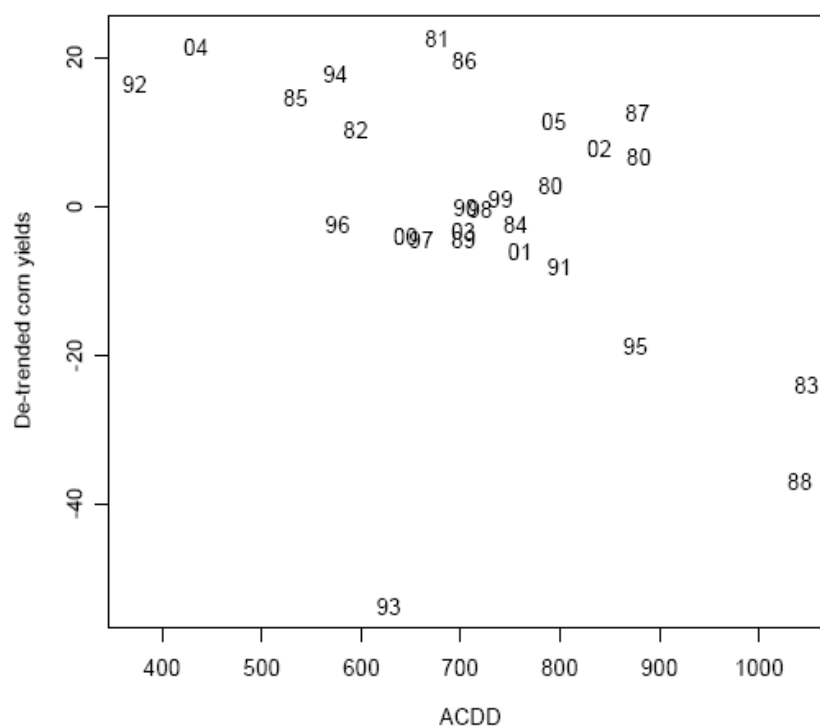


Figure 4.1. Relationship between detrended state average corn yields and state average ACDD in Iowa

Table 4.1 shows the calculated correlation coefficients between ACDD and detrended corn yields for the nine CRDs in Iowa. The correlation analysis excluded data in 1993. The omission of data in 1993 is subjective. It is the judgment of the author that this year was such an outlier that its inclusion would unnecessarily bias the results.

The correlation statistics and the scatter plot indicate that there is a strong negative relationship between ACDD and corn yields when ACDD is above a certain number (around 300). Corn yields were extremely low in years 1983 and 1988 which correspond to two extremely hot years. The strong negative correlation between ACDD and corn yields indicates a possibility of using ACDD as an index in index insurance framework insuring against corn yield reduction.

4.3 Data

Monthly cooling degree days measured at weather stations in Iowa were obtained from NOAA-NCDC. The data used in this research is the accumulated cooling degree days during the summer season from 1980 to 2006. Accumulated cooling degree days during the summer season are calculated as the summation of cooling degree days of during June, July and August. Weather stations which have missing observations in any of the months June, July or August in a year were excluded from the set of observing stations in that year. So, the full data set of Iowa weather stations is reduced to only those

Table 4.1. Correlation between ACDD and corn yields for CRDs in Iowa

CRDs	CRD1	CRD2	CRD3	CRD4	CRD5	CRD6	CRD7	CRD8	CRD9
Correlation	-0.64	-0.54	-0.46	-0.72	-0.57	-0.58	-0.74	-0.65	-0.69

which provide ACDD measure in any of the 27 years. Because of the missing value issue, the number of weather stations providing ACDD records varies by year. There are 53 weather stations which have ACDD records for the entire time period of 27 years. The ACDD data recorded at these 53 stations from 1980 to 2005 is used to fit models. ACDD data in 2006 is used for validation purposes. We call these 53 stations in-sample stations because models are fitted using records measured at those stations. We call all other stations left-aside stations. The number of left-aside stations varied from 59 to 73 from year to year. Left-aside stations are used to test the prediction power of different models through cross-validation approach. Weather stations at which data is collected are shown in Figure 4.2.

The red dot in Figure 4.2 indicates in-sample stations and blue dot indicates left-aside stations. The left-aside stations in a single year are a subset of the left-aside stations shown in this figure. The grid of 53 in-sample stations provides good coverage for the major agricultural areas of Iowa. The geographic coordinates of each of the weather stations in Iowa can be obtained from NOAA-NCDC which is measured in degree of

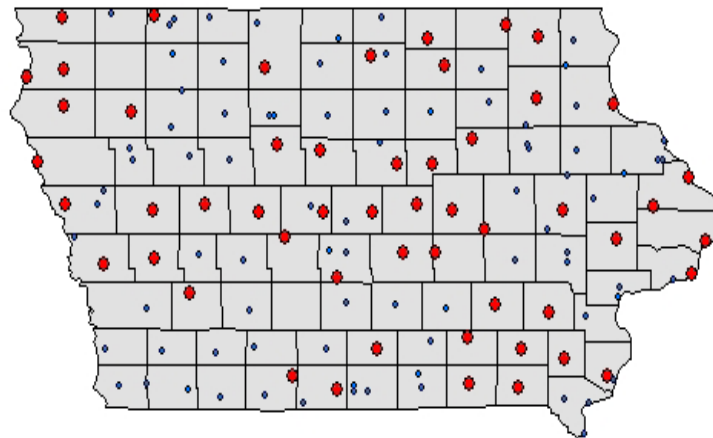


Figure 4.2. Locations of observing stations

latitude and longitude. Latitudes and longitudes for stations are transformed into X and Y coordinates in terms of Euclidian distance from a specific location in Nebraska as the grid origin. The distance between adjacent in-sample weather stations averages 36.8 kilometers, with a maximum (minimum) distance between weather stations 72.4 (24.1) kilometers.

Figure 4.3 maps the means of reported ACDD levels for the counties in which the weather stations are located. The county ACDD means are the average of ACDD over stations in the county over years from 1980 to 2006. The figure shows that the south section of Iowa is the hottest. It gets cooler the more north one goes as expected.

4.4 Methodology

To address spatial basis risk, two sophisticated spatial approaches are employed. One is the less-developed MRF approach. The other is the well-developed geo-statistical approach. In addition, a naive multiple linear regression approach is also applied where

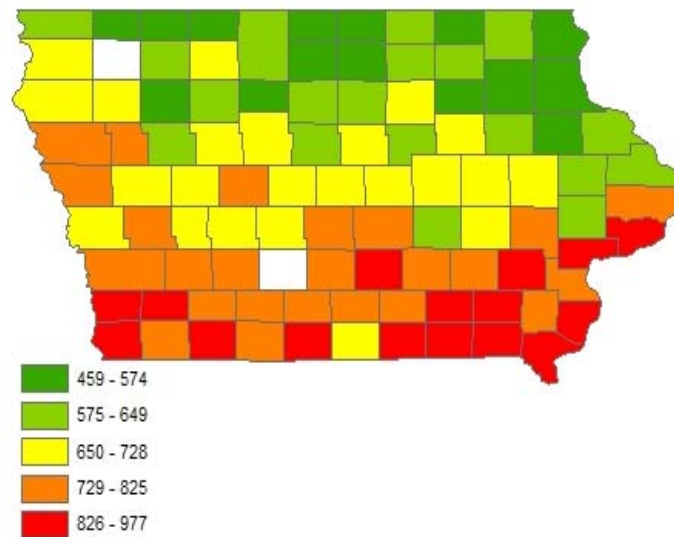


Figure 4.3. Reported county ACDD means (Fahrenheit) in Iowa

no spatial dependence of ACDD is assumed. This section presents the three models and prediction techniques to allow a comparison of the performance of MRF with the two common alternative approaches.

4.4.1 Markov random field model

A Markov random field (MRF) can be defined for any set of random variables defined on a lattice. The fact that the lattice is not necessarily regular makes MRF models suitable for this research where data is gathered at unequally spaced weather stations.

Let $S_i \equiv (u_i, v_i)$ denotes the location of weather station i , where u_i denotes the X coordinate and v_i denotes the Y coordinate in terms of Euclidian distance from the grid origin which is located in northeast corner of Nebraska. The set of random variables considered is denoted as $Y \equiv \{y(s_i, t) : i = 1, \dots, S; t = 1, \dots, T\}$, where $y(s_i, t)$ is the random variable defined at station i in year t . The total number of stations in a given year is denoted by S and total number of years considered is denoted by T . A MRF model requires the specification of conditional distributions for each location conditional on all other locations as (4.2).

$$f_i(y(s_i, t) | y(s_j, t) : j \neq i), \quad i, j = 1, \dots, S \quad (4.2)$$

A MRF model also requires that a neighborhood of observations be defined for each location. In this study, we use the estimated range of spatial correlation to define the neighborhood. The estimate of range is obtained as the minimum prediction mean squared error (PMSE) solution for all left-side locations in year 1980 to 2005. We define neighborhood of station i , N_i , as the set of station j such that $|s_i - s_j|$, the Euclidian distance

between station i and station j , is less than or equal to the estimated range. Neighborhoods must be symmetric, that is, if $i \in N_j$, then $j \in N_i$. Since the lattice considered in this research is irregular, the number of neighborhood stations is different for different stations, varying from 6 to 23 for in-sample stations. The Markov part of the MRF model comes from the assumption that,

$$f_i(y(s_i, t) | y(s_j, t) : j \neq i) = f_i(y(s_i, t) | y(s_j, t) : j \in N_i), \quad i, j = 1, \dots, S \quad (4.3)$$

(4.3) says that, given values at all other locations, the distribution of $y(s_i, t)$ depends only on those values at locations within its neighborhood.

The normality of standard seasonal and monthly indices on US temperatures was analyzed in Jewson (2004) with the conclusion that, for the summer CDD, the normal distribution gives a reasonable fit at almost all locations. This is equivalent to saying that the random variable of interest, ACDD, at any location follows a normal distribution. It is reasonable to assume ACDD for the entire set of locations follows a multivariate normal distribution for this research. If ACDD is multivariate normal, then the distribution of ACDD, conditional on neighborhood observations, and marginal distribution of ACDD will also be normal. Therefore, the conditional densities for each element of Y defined above is given as (4.4).

$$f_i(y(s_i, t) | y(s_k, t) : k \in N_i) = (2\pi\tau_i^2)^{-1/2} \cdot \exp\left[-\frac{1}{2\tau_i^2} \{y(s_i, t) - A_i(y(s_k, t), k \in N_i)\}^2\right] \quad (4.4)$$

where

$$A_i(y(s_k, t), k \in N_i) \equiv E[y(s_i, t) | \{y(s_k, t), k \in N_i\}]$$

$$\tau_i^2 \equiv \text{var}[y(s_i, t) | \{y(s_k, t), k \in N_i\}]$$

Based on the suggestion of Besag (1974) about the necessary structure form for conditional mean, we further model conditional mean as represented by (4.5).

$$A_i(y(s_k, t), k \in N_i) = \alpha_{it} + \sum_k c_{ik} \{y(s_k, t) - \alpha_{kt}\} \quad (4.5)$$

subject to the conditions that $c_{ik}\tau_k^2 = c_{ki}\tau_i^2$ and $c_{ii} = 0$. Formulation (4.4) and (4.5) indicate that the marginal expectation of $y(s_i, t)$ is equal to α_{it} , or, $E(y(s_i, t)) = \alpha_{it}$, and this provides one way to incorporate information on spatially or temporally varying covariates. Namely, covariates can be incorporated into the marginal mean structure. Suppose we have q covariates and we wish to include an intercept term in a linear model. This can be accomplished by taking

$$X_{it} = (1, x_1(s_i, t), \dots, x_q(s_i, t))^T$$

and modeling

$$\alpha_{it} = X_{it}^T \beta$$

or

$$\alpha = X\beta \quad (4.6)$$

Therefore, the joint probability distribution for $Y \equiv \{y(s_i, t) : i = 1, \dots, S; t = 1, \dots, T\}$ is given by

$$Y \sim \text{Gau}(X\beta, (I - C)^{-1} M) \quad (4.7)$$

In (4.7), $X\beta = \{X\beta(s_i, t), i = 1, \dots, S; t = 1, \dots, T\}$, which is the marginal mean vector for in-sample stations. I is the N by N identity matrix where N is equal to 1378, which is calculated as the number of in-sample stations (53) multiplied by the number of years (26) considered in

the model. Since covariates are included in the marginal mean structure, an assumption of constant conditional variance τ^2 is made. M is thus an N by N diagonal matrix with diagonal entries identically equal to τ^2 . C is an N by N matrix with entry of pair-wise dependence parameter. In this way, the symmetric constraint $c_{ik}\tau_k^2 = c_{ki}\tau_i^2$ mentioned before is satisfied.

For the conditional specified model described as above, the large-scale structure is shown by the marginal mean structure, while the small-scale structure is entirely captured in dependence matrix C . The matrix C contains 1378 different values which are too many for unrestricted estimation. In addition, meaningful models need to provide some structure for spatial dependence (e.g. isotropy, directional, etc). In this study, we assumed unidirectional dependence as a function of Euclidian distance. Namely, the spatial dependence does not depend on direction but on distance between the two locations involved. We focus on spatial dependence of ACDD and assume that there is no temporal dependence of ACDD across the historical time period investigated. Therefore, the C matrix is a diagonal block matrix with 53 by 53 matrices on the diagonal.

To model spatial dependence as a decreasing function of distance and as being constant throughout time, we employ the following pair-wise dependence structure

$$c_{ij} = \begin{cases} \eta \left(\frac{1}{d_{ij}} \right)^p & j \in N_i \\ 0 & o.w. \end{cases} \quad (4.8)$$

In (4.8), $d_{ij} = |s_i - s_j|$ is the Euclidian distance between stations i and j ; p is the scale factor of inverse distance so that the scaled distance measure used is appropriate for statistical dependence. η is the constant spatial dependence parameter.

The approach we adopted for estimation is marginal likelihood maximization.

Following the idea of Cressie (1993), combining structure (4.4), (4.5) and (4.6), we have the joint density of elements of Y as (4.9).

$$f(Y | \beta, \tau^2, C) = (2\pi\tau^2)^{-(N/2)} \cdot |I - C|^{1/2} \cdot \exp\left[\left(-\frac{1}{2\tau^2}\right)(Y - X\beta)'(I - C)(Y - X\beta)\right] \quad (4.9)$$

The negative log likelihood is therefore given by

$$L(\beta, \tau^2, C) = \left(\frac{N}{2}\right) \log(2\pi\tau^2) - \frac{\log(|I - C|)}{2} + \frac{1}{2\tau^2} (Y - X\beta)'(I - C)(Y - X\beta) \quad (4.10)$$

In (4.10), $I - C$ is symmetric and positive definite. The negative log likelihood can be minimized applying a two-stage procedure. First, considering η to be fixed, minimization of the negative log likelihood leads to the following

$$\begin{aligned} \hat{\beta} &= (X'(I - C)X)^{-1} X'(I - C)Y \\ \hat{\tau}^2 &= (Y - X\hat{\beta})'(I - C)(Y - X\hat{\beta})/N \end{aligned} \quad (4.11)$$

(4.11) gives the maximum profile likelihood estimates of β and τ^2 . Substituting these estimates back to the negative log likelihood function, the maximum likelihood estimator of η can be obtained by minimizing the negative log profile likelihood (4.12) with respect

$$L(C) = \left(\frac{N}{2}\right) \log((2\pi) + 1) - \frac{\log(|I - C|)}{2} + \frac{N}{2} \log\left[Y'(I - C)\left\{I - X(X'(I - C)X)^{-1} X'(I - C)\right\}Y\right] \quad (4.12)$$

to the unknown η . In the minimization process, the necessary checks on η to ensure positive definite requirement is included. The maximum likelihood estimates of β and τ^2 can be obtained by plugging estimated η back to (4.11). The power of inverse distance p is set to be equal to one in the spatial dependence structure. A profile analysis for p is performed by

setting p as 0.25, 0.5, 0.75 and 1, respectively. The results indicate that the profile likelihood was maximized at $p = 1$.

Models are derived and fit for ACDD data measured at in-sample stations in year 1980 to 2005, a total of 26 years. ACDD records at left-aside locations are used to test the prediction power of the fitted model through cross-validation procedure. The statistic employed to test prediction power of different models is the prediction mean squared error. Let $y(s_l, t)$ denote the ACDD at the left-aside location s_l in year t . Assume that the distribution of $y(s_l, t)$ given $y(s_k, t), k = 1, \dots, 53$, is of the same form as (4.4), namely, we assume that ACDD at in-sample locations and left-aside locations are following the same distribution. Then the optimal predictor is given by (4.13).

$$E[y(s_l, t) | y(s_k, t) : k \in N_l] = A_l(y(s_k, t), k \in N_l) \quad (4.13)$$

(4.13) is the conditional mean of ACDD at location s_l conditional on ACDD of its neighborhood which is formed exclusively by a subset of in-sample locations. The expectation is immediately available from the conditional specified model. Using maximum likelihood estimates as plug-in values, we can obtain predicted values for left-aside location s_l as (4.14).

$$E[y(s_l, t) | y(s_k, t) : k \in N_l] = X_l \hat{\beta} + \sum_{k \in N_l} c_{lk} (y(s_k, t) - X_k \hat{\beta}) \quad (4.14)$$

Replicating this process, we can get predicted value of ACDD at all left-aside locations across 26 years. Finally, the PMSE can be calculated according to formulae (4.15).

$$\begin{aligned}
 PMSE &= E \left[\left(Y_{left-side} - \hat{Y}_{left-side} \right)^2 \right] \\
 &= \frac{1}{M} \sum_{l=1}^M \left[\left(y(s_l, t) - \hat{y}(s_l, t) \right)^2 \right]
 \end{aligned} \tag{4.15}$$

where $\hat{y}(s_l, t) = E \left[y(s_l, t) \mid \{y(s_k, t), k \in N_l\} \right]$ and M is the total number of left-side locations in year 1980 to 2005.

4.4.2 Multiple linear regression

The naive method to model this ACDD data is to fit a multiple linear regression and predict for left-side locations across years assuming there is no underlying spatial or temporal dependence at all. The AIC criterion is adopted to choose from candidate multiple linear models. The chosen model is represented as (4.16)

$$ACDD_{it} = \beta_0 \cdot avg_acdd_t + \beta_1 x_i + \beta_2 y_i + \beta_3 x_i^2 + \beta_4 x_i y_i + \beta_5 y_i^2 + \varepsilon_{it} \tag{4.16}$$

In (4.16), t indexes year, $t = 1980, \dots, 2005$; i indexes in-sample weather stations, $i = 1, \dots, 53$; $ACDD_{it}$ is the accumulated cooling degree days during the summer season of station i in year t ; avg_acdd_t is the average ACDD in year t in Iowa; x_i and y_i are the X coordinate and Y coordinates of station i , which are constant across years. Similarly, x_i^2 , $x_i y_i$ and y_i^2 are the second-order polynomial terms in X and Y coordinates of station i . Table 4.2 reports the coefficient estimates of multiple linear regression. The number in the parenthesis is the standard error of the corresponding estimate.

All the parameter estimates are significant at the 0.1% significance level. The 98.65% of R^2 implies that 98.65% of the variability of ACDD across in-sample stations across years can be explained by the model. It is easy to understand that R^2 is as

Table 4.2. Regression coefficient estimates

$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	R^2
1.0257	0.2011	0.4746	0.0005	-0.0024	-0.0014	0.9865
(-0.0143)	(-0.0641)	(-0.0963)	(-0.0001)	(-0.0001)	(-0.0002)	---

high as 0.9865 since we put the yearly average ACDD in the model as a covariate which is expected to have a very high correlation with ACDD at each weather station.

The prediction based on this multiple linear regression approach is simple. For any left-aside location, if the corresponding yearly average ACDD and its X and Y coordinates are known, the predicted ACDD at that location can be obtained simply by plugging them in the fitted linear model. The PMSE for all left-aside locations can be calculated by formulae (4.17).

$$\begin{aligned}
 PMSE &= E \left[\left(Y_{left-aside} - \hat{Y}_{left-aside} \right)^2 \right] \\
 &= \frac{1}{M} \sum_{l=1}^M \left[\left(y(s_l, t) - \hat{y}(s_l, t) \right)^2 \right]
 \end{aligned} \tag{4.17}$$

where $\hat{y}(s_l, t) = X_l \hat{\beta}$ and M is the total number of left-aside locations in year 1980 to 2005.

4.4.3 Geo-statistical model

Different from MRF approach, the standard geo-statistical approach does not assume any distribution for the data. Instead, a spatial variogram is employed to quantify spatial dependence. For locations s_i and s_j and associated random variables $z(s_i)$ and $z(s_j)$, the variogram is defined as (4.18).

$$\gamma(s_i - s_j) \equiv \frac{1}{2} \text{var} \{ z(s_i) - z(s_j) \} \quad \text{all } s_i, s_j \in D \tag{4.18}$$

where D is the entire set of weather stations. To calculate sample variogram, suppose that

$E\{z(s)\} = \mu; \forall s \in D$, then we would have

$$\gamma(s_i - s_j) = \frac{1}{2} E[z(s_i) - z(s_j)]^2 \quad (4.19)$$

(4.19) suggests using the average of squared differences in data values to represent $\gamma(\cdot)$. For a

given displacement h , if we have observations from a set of pairs of

locations $N(h) \equiv \{(s_i, s_j) : s_i - s_j = h; i, j = 1, \dots, n\}$, a sample variogram for this displacement

would be

$$\gamma(h) = \frac{1}{2|N(h)|} \sum_{N(h)} \{z(s_i) - z(s_j)\}^2 \quad h \in R^d \quad (4.20)$$

In (4.20), $|N(h)|$ is the number of pairs in the set $N(h)$. Without repeated observations at the

same displacement, we will not have sets $N(h)$. In fact, typically we will not have more than

a few pairs of observations with the same displacement. To deal with this problem, we use

the idea of tolerance region. Suppose, in a given set of data, there are m distinct

displacement values. Choose k of these values $h(l), l = 1, \dots, k$, and define tolerance regions

as $T(h(l)), l = 1, \dots, k$. Then the set of paired locations with distance within the corresponding

tolerance region can be defined as

$$N(h(l)) \equiv \{(s_i, s_j) : s_i - s_j \in T(h(l))\}, l = 1, \dots, k.$$

We will consider two of the most common variogram models in this analysis. Both models make the assumption that displacement between two locations $(s_i - s_j)$ enters the

variogram only in terms of distance $d_{ij} = |(s_i - s_j)|$. The first is the exponential variogram

model

$$\gamma(h | \theta) = \begin{cases} 0 & h = 0 \\ a + b(1 - \exp(-|h|/c)) & h \neq 0 \end{cases} \quad (4.21)$$

where $\theta = (a, b, c)$, with $a \geq 0, b \geq 0$ and $c \geq 0$. The second is the spherical variogram model

$$\gamma(h | \theta) = \begin{cases} 0 & h = 0 \\ a + b(1.5 \cdot (|h|/c) - 0.5 \cdot (|h|/c)^3) & 0 < h \leq c \\ a + b & h \geq c \end{cases} \quad (4.22)$$

where $\theta = (a, b, c)$, with $a \geq 0, b \geq 0$ and $c \geq 0$. Both variogram models are characterized by

the same set of parameters as follows. The range c is the distance at which the variogram

reaches a maximum (if it does). The interpretation of the range is that it is the distance

beyond which $z(s+h)$ and $z(s)$ are no longer correlated. The sill $a+b$ is the value of the

variogram when evaluated at any distance greater than or equal to the range. The sill is

usually a constant number. The nugget a presents micro-scale variance. Clearly, $\gamma(0) = 0$ if

$|h|=0$. But it may not be true that $\gamma(h/\theta) \rightarrow 0$ as $|h| \rightarrow 0$. If $\gamma(h/\theta) \rightarrow a$ as $|h| \rightarrow 0$, then a

is called the nugget. The nugget is how much of the variation of the response variable cannot

be explained spatially.

The prediction method in geo-statistical approach is referred as kriging. Different types of kriging can be chosen according to the underlying assumption for the stochastic process.

Ordinary kriging is chosen in this study which assumes that the underlying process is

intrinsically stationary. A process is said to be intrinsically stationary if it is of constant mean

and constant variogram at a certain distance. In practice, the assumption that variogram is a

function of distance alone is difficult to verify---it remains an assumption (Cressie, 1993).

Therefore, modeling and prediction are carried out based on residuals instead of the original

ACDD data measured at in-sample locations which satisfy the constant mean assumption.

Given the fitted model from multiple linear regression approach, ACDD data for in-sample locations can be detrended by subtracting the fitted values from the original ACDD values to get ACDD residuals. The residuals satisfied the stationary assumption required for empirical variogram estimation. We assume unidirectional spatial dependence or isotropy as we did in MRF modeling, which means that displacement between two locations enters the variogram only in terms of distance, but not of direction. Empirical spatial variograms at different distances for each of the 26 years from 1980 to 2005 is estimated using a package called “geoR” associated with R software. The maximum distance we considered for the range is 400 kilometers which turns out to be much greater than the estimated range. For each year, we can get a vector of spatial variograms with each element as the value of the spatial variogram associated with a certain distance defined as the midpoint value of the corresponding bins.

Generally, variogram models are fitted to data recorded at the same year or same day. For multiple-year data as in this study, one could fit variogram models for each year. But, it is difficult to choose among the fitted models when they are quite different from year to year. We address this difficulty by stacking the 26 estimated variogram values across years for each of the bins and fit a variogram model to the stacked data. The obtained estimate of range is thus an expectation of spatial correlated range across years. However, different fitted models can be obtained by varying the size of the bins or the midpoint value of first bin. The estimate of spatial correlated range varies from about 90 to 150 kilometers depending on the model selected. This is the nature of variogram model fitting and thus the big drawback of geo-statistical approach when multiple years of data are involved. Therefore, instead of searching for the best model using standard geo-statistical approach, which is actually

impossible because of the multiple-year data explored in this study, we decide to find the fitted variogram model of which the estimated range is close enough to the one obtained from MRF model. A fitted spherical model is obtained as shown in Table 4.3. With the parameter estimates of nugget a , sill $a + b$ and range c , we can predict for left-aside locations across years using ordinary kriging.

It is known that under squared error loss, the optimal predictor is the conditional mean which can be calculated accordingly provided with probability distribution (Casella and Berger, 2002). But, geo-statistical analysis does not assign any probability distribution. Instead, it provides the linear, unbiased predictor for left-aside location s_l in year t as the linear combination of residuals at in-sample locations as (4.23).

$$z(s_l, t) = \lambda \cdot Z = \sum_{i=1}^{53} \lambda_i z(s_i, t) \quad (4.23)$$

Unbiasedness requires that $E(z(s, t)) = \mu$, which is equivalent to require that $\sum_{i=1}^{53} \lambda_i = 1$.

Thus, minimization of the PMSE becomes minimization of

$$E \left[z(s_l, t) - \sum_{i=1}^{53} \lambda_i z(s_i, t) \right]^2 - m \cdot \left\{ \sum_{i=1}^{53} \lambda_i - 1 \right\}$$

where m is a lagrange multiplier due to the constraint $\sum_{i=1}^{53} \lambda_i = 1$. Solving this minimization

Table 4.3. Spherical model parameter estimates

parameter	Estimate	Std. Error	t value
a	5614.84	500.28	11.22
b	1908.6	509.29	3.75
c	133.69	51.81	2.58

problem yields the following:

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{53} \\ m \end{pmatrix} = \begin{pmatrix} \gamma(s_1 - s_1), \gamma(s_1 - s_2), \dots, \gamma(s_1 - s_{53}), 1 \\ \gamma(s_2 - s_1), \gamma(s_2 - s_2), \dots, \gamma(s_2 - s_{53}), 1 \\ \cdot \\ \cdot \\ \cdot \\ \gamma(s_{53} - s_1), \gamma(s_{53} - s_2), \dots, \gamma(s_{53} - s_{53}), 1 \\ 1, \quad 1, \dots, \quad 1, 0 \end{pmatrix}^{-1} \begin{pmatrix} \gamma(s_l - s_1) \\ \gamma(s_l - s_2) \\ \cdot \\ \cdot \\ \cdot \\ \gamma(s_l - s_{53}) \\ 1 \end{pmatrix}$$

or

$$\lambda = \Gamma^{-1} \gamma \quad (4.24)$$

In (4.24), λ is the vector containing linear coefficient estimates together with the lagrange multiplier estimate. The entry $\gamma(s_i - s_j)$ in the Γ matrix are estimated variograms between in-sample location s_i and s_j , $i, j = 1, \dots, 53$. The entry $\gamma(s_l - s_i)$ in the γ vector are estimated variograms between left-aside location s_l and in-sample location s_i , $i = 1, \dots, 53$. All the estimated variograms are calculated according to the fitted spherical model with estimated nugget, sill and range as the plug-in values.

A linear combination of the 53 in-sample residuals with coefficients calculated as above gives the predicted residual of the left-aside location s_l as (4.25).

$$\hat{z}(s_l, t) = \sum_{i=1}^{53} \hat{\lambda}_i z(s_i, t) \quad (4.25)$$

This $\hat{z}(s_l, t)$ is only the predicted value for the residual at the location s_l in year t . To predict ACDD we need to add this predicted residual back to the estimated trend for that location in year t which can be calculated according to the fitted multiple linear regression (4.16) by

plugging in the coefficients estimates together with the X, Y coordinates and corresponding yearly average ACDD. Similarly, prediction of ACDD at other left-aside locations across years can be derived. Provided with predicted ACDD at all left-aside location, the PMSE desired is simply

$$\begin{aligned} PMSE &= E \left[\left(Y_{left-aside} - \hat{Y}_{left-aside} \right)^2 \right] \\ &= \frac{1}{M} \sum_{l=1}^M \left[\left(y(s_l, t) - \hat{y}(s_l, t) \right)^2 \right] \end{aligned} \quad (4.26)$$

In (4.26), $\hat{y}(s_l, t) = X_l \hat{\beta} + \hat{z}(s_l, t)$ and M is the total number of left-aside locations in year 1980 to 2005.

4.5 Prediction results and model comparisons

For problems where the primary objective is spatial prediction, the criterion for model assessment is cross-validation. We conducted a cross-validation comparison of the linear model, geo-statistical model and MRF model using ACDD data. The PMSE of the left-aside locations across years applying the three methods are given in Table 4.4.

It is clear from Table 4.4 that the geo-statistical approach offers improvement over multiple linear regression approach in terms of prediction. The prediction error is reduced by 9.5%. The improvement of MRF approach over geo-statistical approach is smaller but positive, suggesting that the MRF approach to spatial prediction may be promising. Multiple linear regression is not a good method to model ACDD if ACCD is spatially correlated

Table 4.4. PMSE of models of ACDD

Model	Linear	Geo-statistics	MRF
PMSE	216643.6	196157.8	191211.7

because it does not take into consideration of the spatial dependence of ACDD, whereas the geo-statistical approach and MRF approach do.

For ordinary kriging, the stationary assumption must be satisfied. Therefore, modeling and prediction are based directly on residuals which satisfy the constant mean assumption. The MRF approach requires parametric specification of the distribution for variables of interest and has no stationary assumption. Instead of working on residuals, the modeling and prediction are carried out by considering jointly the large-scale structure and small-scale structure which is residuals from the large-scale structure. One of the reasons that MRF approach out-performed the geo-statistical approach is that the large-scale structure is estimated taking into consideration the variability in the small-scale structure. Namely, the large-scale structure and the small-scale structure are estimated jointly. Different from ordinary kriging where prediction for a certain location relies on the entire data points, for the MRF approach, the prediction for certain location only depends on locations within its neighborhood. If it is true that given values at locations in the neighborhood, there is no dependence on other locations, then MRF model will perform better because prediction is based on local information which is believed to be more relevant. On the contrary, the prediction of geo-statistical approach is based on all in-sample locations, some of which may not be relevant at all. The nature of geo-statistical model estimation when multiple years or days are involved makes it a drawback of that approach. While, for parameter estimation in the MRF approach, the likelihood of the multiple-year data are maximized assuming a normal distribution as shown by (4.7). The automated nature of estimation with MRF models make it easier to model multiple years of data.

4.6 Index insurance contract and its pricing

A temperature insurance policy is proposed to protect farmers against the risk of corn yield reduction due to high temperature based on cooling degree days accumulated during the summer months of June, July and August. The reason that we choose a seasonal contract rather than a strip of monthly contracts is that monthly temperatures are typically auto-correlated (Jewson and Brix, 2005). Using a strip of monthly contracts may lead to an over-hedging problem. In addition, transaction costs associated with a strip of monthly contracts is higher than a single seasonal contract. A linear regression relates ACDD to time indicate there is no significant warming trend in any of the nine CRD in Iowa. Figure 4.4 shows the average ACDD versus time from 1980 to 2006 in Iowa. The results are consistent with Jewson and Brix's findings of weak or no trend in summer temperature in the United State.

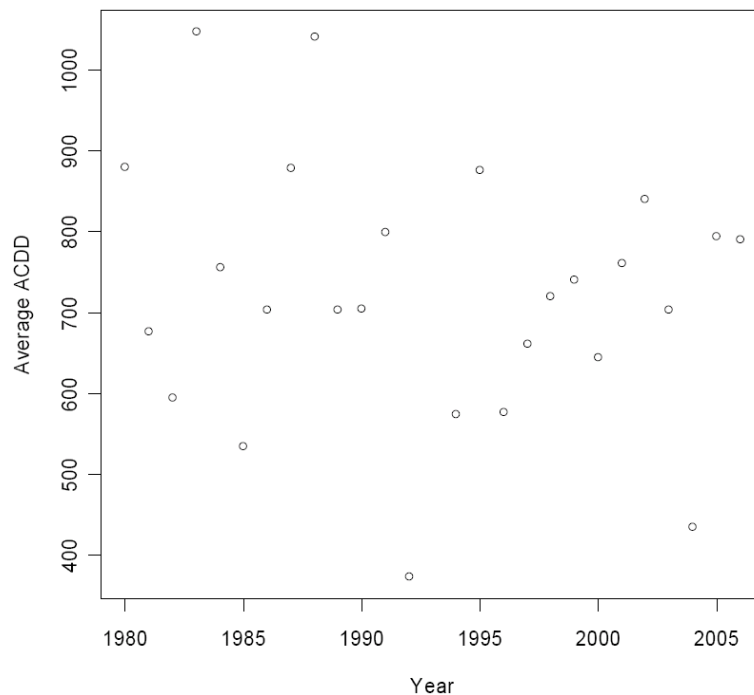


Figure 4.4. Average ACDD over time from 1980 to 2006 in Iowa

Therefore, the ACDD guarantee is set as the 26-year average of ACDD for an insured location. The ACDD guarantee is obtained by taking the average of either the recorded ACDD over 26 years for locations where weather station data are available or the interpolated ACDD for locations where a weather station is not available. The liability value is determined by expected revenue which is the product of expected corn price, expected corn yield, insured acres and coverage level. The price can be set as the CBOT average December future price quoted in February. The expected yield can be set as the predicted trend yield.

The indemnity function takes the form of an exotic call option on the value of actual ACDD. Under the proposed plan, insured customers will receive a claim payment if the actual ACDD index is higher than the ACDD strike value. Put in mathematical form:

$$liability = P_F \cdot Y_T \cdot A \cdot C, \text{ and}$$

$$indemnity = \max \left\{ 0, \min \left[L \times \left(F \times \frac{ACDD_A - ACDD_S}{ACDD_G} \right), L \right] \right\} \quad (4.27)$$

In (4.27), P_F : CBOT average December corn future price quoted in February

Y_T : trend yield

A : insured acreage

C : coverage level ($C \in [0,1]$)

L : liability

F : indemnity factor

$ACDD_A$: the actual ACDD recorded or interpolated (predicted)

$ACDD_S$: strike value of ACDD

$ACDD_G$: the guaranteed ACDD

Following the idea of Paulson (2006), the indemnity factor, F , is estimated for each CRD in Iowa through a simple linear regression which relates corn yields to ACDD index. The left-hand side variable Y_R is the ratio of the actual yield to the trend yield. The right-hand side variable $ACDD_R$ is the ratio of actual ACDD to the 26-year average in each CRD. In this way, the corresponding estimated coefficient can be interpreted as the percentage of yield shortfall below the trend yield due to one percent increase of ACDD above the long-term average. The linear model is represented as (4.28).

$$Y_R = \hat{\beta}_0 + \hat{\beta}_1 \cdot ACDD_R + \varepsilon \quad (4.28)$$

Table 4.5 reports the regression estimates for each of the nine CRD in Iowa. The number in parenthesis is the standard error of corresponding coefficient estimate. The results indicate that there is a fairly strong negative relationship between corn yield and ACDD with a one percent increase in the ACDD above the 26-year average resulting in between a 0.2 percent decrease in corn yields below the trend yield (in the North Central CRD) and a 0.72 percent decrease in corn yields (in the Southeast). The R^2 can be interpreted as the portion of total yield variability explained by this specific weather event defined as ACDD. The coefficient estimate $\hat{\beta}_1$ is taken as the indemnity factor for the corresponding

Table 4.5. Regression estimates for each of the nine CRD in Iowa

CRD	NW	NC	NE	WC	C	EC	SW	SC	SE
$\hat{\beta}_1$	-0.26 (0.06)	-0.2 (0.06)	-0.23 (0.09)	-0.36 (0.07)	-0.29 (0.08)	-0.43 (0.12)	-0.50 (0.09)	-0.53 (0.13)	-0.72 (0.15)
R^2	0.41	0.29	0.21	0.52	0.33	0.34	0.55	0.42	0.48

CRD. Thus, for every one percent increase in ACDD above the 26-year average, the policy will pay a different percentage of the liability value in different CRDs, varying from 0.2 percent to 0.72 percent.

To rate the proposed insurance policy, Monte Carlo simulation technique is employed. In order to apply Monte Carlo simulation, the distribution of the underlying temperature index needs to be determined. The study of normality of standard seasonal and monthly indices on US temperatures conducted by Jewson (2004) suggests that for CDD index based on the individual months, the normal distribution does not do particularly well overall, but for a multiple-month CDD index, the normal distribution gives a reasonable fit at almost all locations. Therefore, we assume a normal distribution for the underlying ACDD index.

In the United States, climate autocorrelation last up to at least six months, principally due to the effects of El Nino Southern Oscillation (ENSO) (Jewson, 2004). This means that historical indices for contracts of longer than around six months cannot really be considered independent. Since our contract is based on seasonal index, we assume independence of estimated histories of ACDD from year to year.

Given the normality assumption, independence assumption, and the fact of no warming trend, sample means and variances are taken as the estimates of the means and variances of the normal distribution for ACDD variable defined at county center points. Given the sample mean and variance for each county, 5000 ACDD random normal deviates are obtained for each reference point. The insurance policy is rated in terms of premium rate, namely, premium charged per dollar value of liability. According to the indemnity structure of the insurance contract, 5000 loss costs are calculated and the unconditional average of these 5000 loss costs gives the break-even premium rate for the policies written on the county

reference points. Table 4.6 reports the premium rates at different strike levels which are chosen as 100%, 115%, 130% and 145% of the guaranteed ACDD level with the highest strike level matching the average historical extremes in ACDD.

Iowa premium rates average 3.56 percent at 100% strike level, and 1.40 percent, 0.43 percent and 0.11 percent at 115%, 130% and 145% strike levels, respectively. The premium rates are not restrictive, especially at high strike levels. For 100% strike level, the premium rates average 3.56 percent with a standard deviation of 1.43 percent. The maximum premium rate is 10.60 percent at the Keokuk County reference point in Southeast Iowa and the minimum value is 1.93 percent at the Wright County reference point in North Central Iowa. Figure 4.5 maps the premium rate levels at a 100% strike level. In general, premium rate levels are the lowest in the North Central and Northeast section of the state and are the highest in the Southeast section of the state.

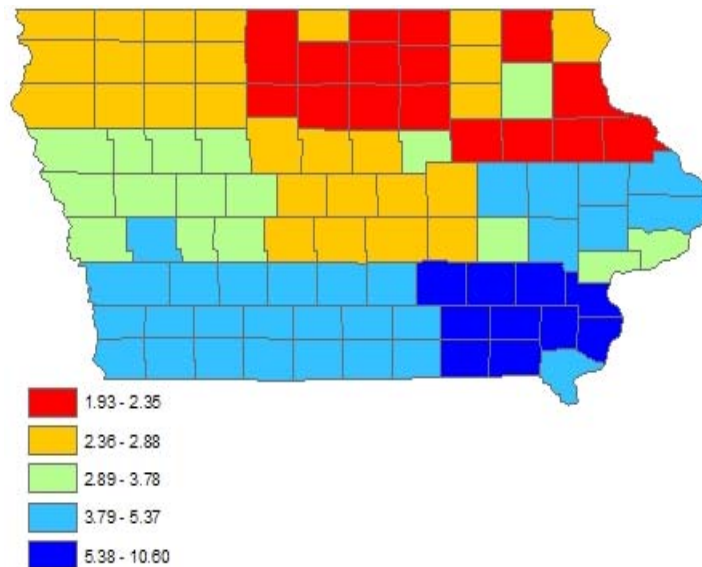


Figure 4.5. Map of Iowa premium rates at 100% strike level

Table 4.6. Iowa county premium rates at various strike levels (%)

County	CRD	100%	115%	130%	145%	County	CRD	100%	115%	130%	145%
Buena Vista	NW	2.48	1.02	0.33	0.08	Hardin	C	2.70	1.07	0.33	0.07
Cherokee	NW	2.42	0.93	0.27	0.06	Jasper	C	2.70	1.07	0.32	0.07
Clay	NW	2.55	1.07	0.35	0.09	Marshall	C	2.60	1.00	0.29	0.07
Dickinson	NW	2.71	1.21	0.44	0.12	Polk	C	2.53	0.94	0.26	0.05
Emmet	NW	2.85	1.31	0.49	0.15	Poweshiek	C	2.70	1.03	0.31	0.07
Lyon	NW	2.61	1.10	0.37	0.09	Story	C	2.64	1.00	0.29	0.06
O'Brien	NW	2.58	1.08	0.35	0.09	Tama	C	2.84	1.19	0.39	0.10
Osceola	NW	2.63	1.14	0.39	0.10	Webster	C	2.56	1.00	0.29	0.06
Palo Alto	NW	2.61	1.12	0.39	0.11	Benton	EC	4.05	1.62	0.51	0.11
Plymouth	NW	2.38	0.93	0.27	0.06	Cedar	EC	4.40	1.95	0.69	0.20
Pocahontas	NW	2.49	1.02	0.33	0.08	Clinton	EC	3.92	1.45	0.39	0.09
Sioux	NW	2.41	0.94	0.28	0.06	Iowa	EC	3.78	1.42	0.40	0.07
Butler	NC	2.03	0.86	0.29	0.08	Jackson	EC	4.03	1.64	0.49	0.11
Cerro Gordo	NC	2.06	0.91	0.34	0.09	Johnson	EC	3.95	1.52	0.43	0.09
Floyd	NC	2.09	0.94	0.35	0.09	Jones	EC	4.12	1.67	0.53	0.13
Franklin	NC	1.93	0.78	0.26	0.07	Linn	EC	4.09	1.63	0.48	0.10
Hancock	NC	2.15	0.97	0.36	0.11	Muscatine	EC	3.61	1.27	0.31	0.05
Humboldt	NC	2.02	0.85	0.28	0.07	Scott	EC	3.74	1.30	0.32	0.05
Kossuth	NC	2.12	0.96	0.35	0.10	Adair	SW	4.30	1.56	0.40	0.07
Mitchell	NC	2.27	1.07	0.43	0.15	Adams	SW	4.28	1.54	0.43	0.08
Winnebago	NC	2.44	1.20	0.51	0.18	Cass	SW	4.29	1.53	0.38	0.06
Worth	NC	2.27	1.09	0.43	0.13	Fremont	SW	4.19	1.45	0.35	0.05
Wright	NC	1.93	0.80	0.27	0.07	Mills	SW	4.13	1.44	0.37	0.06
Allamakee	NE	2.88	1.46	0.63	0.23	Montgomery	SW	4.23	1.46	0.35	0.06
Black Hawk	NE	2.23	0.92	0.30	0.07	Page	SW	4.17	1.48	0.38	0.07
Bremer	NE	2.41	1.04	0.36	0.10	Pottawattamie	SW	4.26	1.46	0.32	0.04
Buchanan	NE	2.27	0.96	0.32	0.08	Taylor	SC	4.40	1.58	0.40	0.06
Chickasaw	NE	2.51	1.15	0.43	0.13	Appanoose	SC	4.38	1.51	0.36	0.05
Clayton	NE	2.35	1.02	0.36	0.10	Clarke	SC	4.56	1.64	0.44	0.08
Delaware	NE	2.24	0.93	0.31	0.08	Decatur	SC	4.90	1.92	0.56	0.12
Dubuque	NE	2.24	0.92	0.30	0.08	Lucas	SC	5.37	2.27	0.77	0.20
Fayette	NE	3.24	1.80	0.88	0.38	Madison	SC	4.50	1.60	0.42	0.07
Howard	NE	2.59	1.27	0.53	0.18	Marion	SC	4.62	1.70	0.47	0.09
Winneshiek	NE	2.34	1.01	0.35	0.10	Monroe	SC	4.58	1.65	0.41	0.08
Audubon	WC	2.98	1.02	0.25	0.05	Ringgold	SC	4.69	1.73	0.48	0.09
Calhoun	WC	3.25	1.23	0.34	0.06	Union	SC	4.65	1.71	0.43	0.06
Carroll	WC	2.94	1.00	0.23	0.02	Warren	SC	4.66	1.70	0.44	0.08
Crawford	WC	3.06	1.11	0.29	0.05	Wayne	SC	4.58	1.62	0.41	0.07
Greene	WC	3.04	1.05	0.25	0.04	Davis	SE	6.33	2.42	0.68	0.13
Guthrie	WC	3.00	1.06	0.26	0.04	Des Moines	SE	5.69	1.86	0.41	0.06
Harrison	WC	2.95	0.99	0.22	0.03	Henry	SE	6.30	2.33	0.61	0.10
Ida	WC	3.11	1.13	0.30	0.06	Jefferson	SE	6.71	2.72	0.82	0.16
Monona	WC	2.97	0.98	0.21	0.03	Keokuk	SE	10.6	5.96	2.95	1.30
Sac	WC	3.24	1.20	0.32	0.06	Lee	SE	5.35	1.63	0.32	0.03
Shelby	WC	4.88	2.61	1.24	0.51	Louisa	SE	5.66	1.92	0.46	0.07

Table 4.6. (continued)

County	CRD	100%	115%	130%	145%	County	CRD	100%	115%	130%	145%
Woodbury	WC	3.13	1.11	0.3	0.06	Mahaska	SE	6.26	2.27	0.59	0.12
Boone	C	2.57	0.95	0.26	0.05	Van Buren	SE	5.93	2.08	0.51	0.09
Dallas	C	2.49	0.89	0.23	0.05	Wapello	SE	6.33	2.25	0.56	0.1
Grundy	C	3.07	1.39	0.5	0.14	Washington	SE	6.37	2.43	0.68	0.15
Hamilton	C	2.62	1	0.27	0.05						

4.7 Loss adjustment and historical analysis

The proposed insurance contract can be marketed in March of a year and loss adjustment can be carried out in November of that year when the actual temperature index value is available for locations which have weather stations. Taking 2006 as an example, Markov random field interpolation technique is applied to interpolate ACDD for the 99 county reference points. Then the percent loss for each reference point is calculated based on the interpolated ACDD history at the reference point. Unlike traditional crop insurance where farmers need to prove that damage occurred on his/her farms or county need to prove damage in the case of area-based insurance products, no proof is needed on the insured side under the proposed index insurance design. In addition, insurance companies do not need to adjust purchaser-specific crop loss, which is expensive and contains an element of subjectivity that growers seldom appreciate (Richards et al., 2004). Figure 4.6 maps the calculated percent loss at 100% strike level for each county reference point based on the interpolated ACDD in 2006. The results indicate that most county reference points have accumulated cooling degree days above the corresponding 26-year guarantee. Thus, indemnities are triggered for those reference points. The calculated percent losses vary from 0 to 13.78 with an average 3.35.

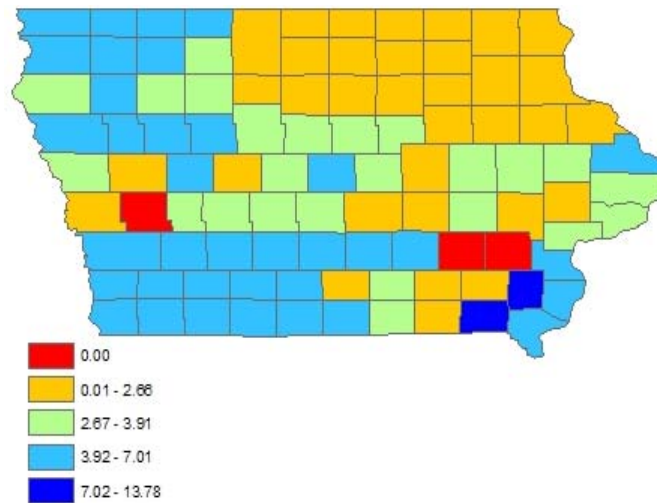


Figure 4.6. Map of Iowa 2006 percent losses at 100% strike level

The loss adjustment procedure is replicated for all contract years from 1980 to 2005. Figure 4.7 to Figure 4.11 maps the calculated percent loss at 100% strike level for each county reference point based on interpolated ACDD in year 1988, 1983, 1981, 2004 and 1993, respectively. The reasons that we are particularly interested in these five years are as follows. 1983 and 1988 are two of the lowest yielding and hot years when corn yields were about 25% and 35% below the historical trend yield average across the state of Iowa. Thus the insurance policy should pay off during these years. 1981 and 2004 are characterized as two extremely mild years when the corn yields are about 20% and 13% above the historical trend yield average across the state of Iowa. 1993 was chosen because corn yields in that year are the lowest in the time period investigated (40% below historical trend yield) due to the extreme wet weather instead of temperature factor in that year.

Figures 4.7 and 4.8 show that indemnity payments are triggered in all counties in 1988 and 1983 with lower percent loss values occurring in north sections of the state and higher percent loss values occurred in south section, especially southeast section of the state. As

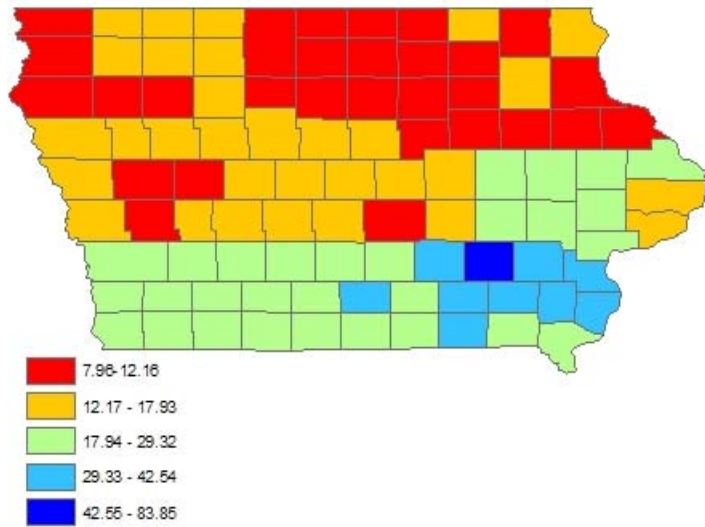


Figure 4.7. Map of Iowa percent losses at 100% strike level in 1988

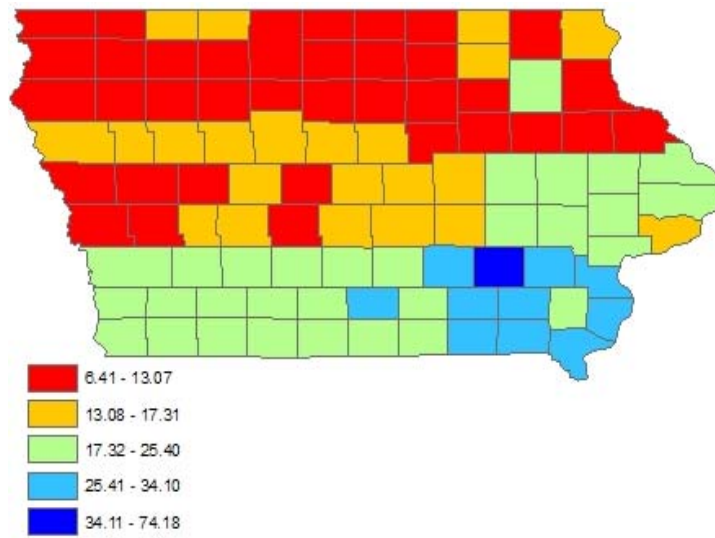


Figure 4.8. Map of Iowa percent losses at 100% strike level in 1983

expected, the percent loss values are higher in 1988 which vary from 7.96 to 83.85 than those in 1983 which vary from 6.41 to 74.18. The figures also show that higher percent loss values occurred in a larger area in 1988 than in 1983.

Figure 4.9 maps the percent loss at 100% strike level in 1981. Indemnity payments are triggered in only a handful of counties. As expected, the percent loss values are zero or small

positive numbers in almost all counties since the year 1981 is a mild year. For counties where losses occur, the percent loss values range from 0.97 percent in Webster County to 11.35 percent in Keokuk County. Figure 4.10 maps the percent loss at 100% strike level in 2004 which is characterized as an extremely mild year. The figure clearly shows that the

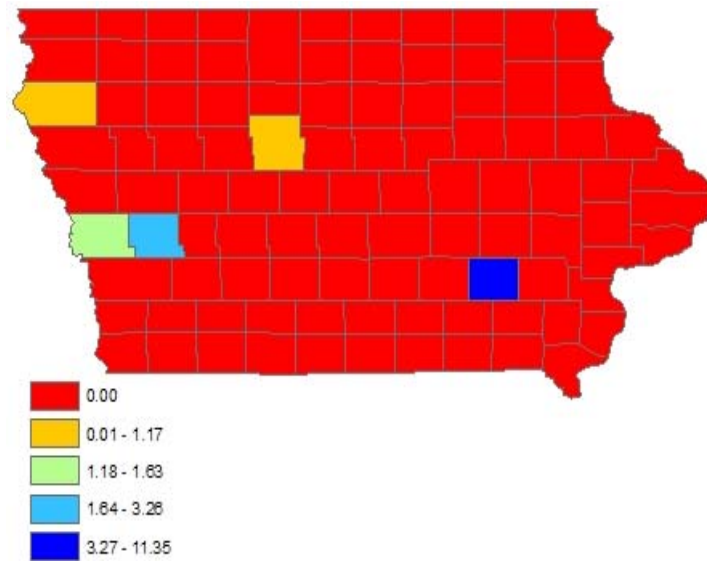


Figure 4.9. Map of Iowa percent losses at 100% strike level in 1981

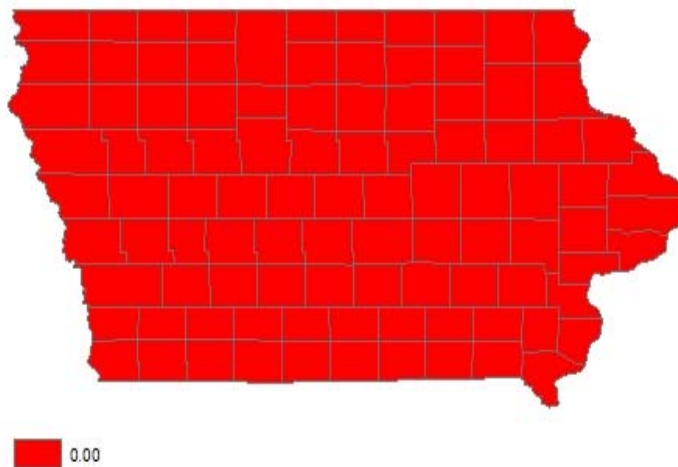


Figure 4.10. Map of Iowa percent losses at 100% strike level in 2004

interpolated ACDD is lower than the 26-year average in all counties and thus no indemnity payment is triggered.

Figure 4.11 maps that percent loss at 100% strike level in 1993 which is characterized by the lowest corn yield in the historical time period investigated in this study. Although the corn yields are extremely low, the figure indicates that no indemnity is triggered in most of the counties. The reason is that the low yields are caused by the extremely wet weather of that year instead of heat stress which is the very risk exposure the proposed insurance policy insuring against. The performance of the insurance policy in this year thus shows it's the limitation with regard to insuring against production risk. It does not provide protection against corn yield reduction due to other reasons than high temperature.

Figure 4.12 shows the loss ratios at the 100% strike level for the 99 counties in Iowa from year 1980 to 2006 except for 1993. The loss ratio is calculated as the ratio of

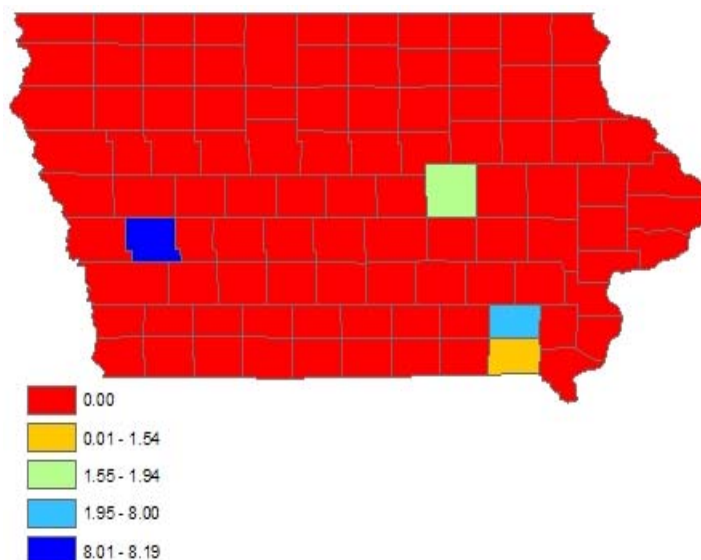


Figure 4.11. Map of Iowa percent losses at 100% strike level in 1993

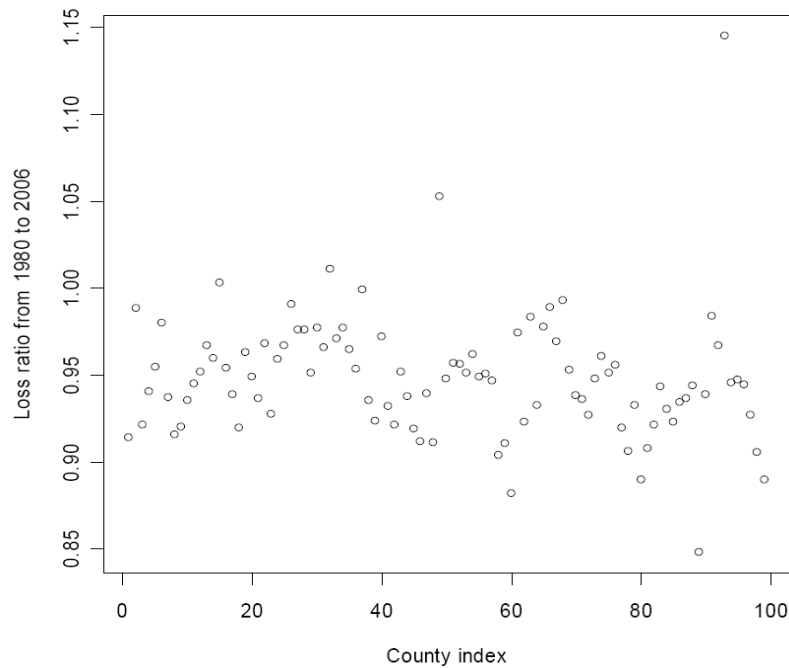


Figure 4.12. Average county loss ratio at 100% strike level in Iowa

indemnity rate to the premium rate, which should average one over time if the policy is actuarially fair. Provided with the interpolated ACDD histories, the indemnity rate or the indemnity per dollar of liability was calculated for each county reference point according to the indemnity structure. The results indicate that the policy is actuarially fair because the historical loss ratio averages 0.95 across counties over the time period investigated which is very close to one. A loss ratio of one is expected for a hypothetical time period which is long enough.

Figure 4.13 shows the scatter plot of the average CRD percent loss cost against average CRD percent yield loss from 1980 to 2006. The percent loss cost for each county is calculated according to the indemnity structure. The average CRD percent loss cost is the average of percent loss cost over a CRD. The average CRD percent yield loss is the

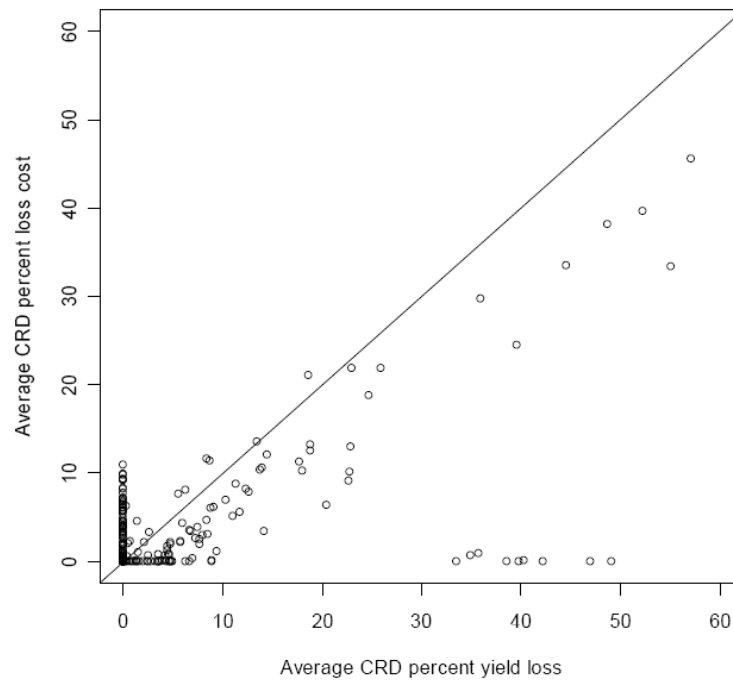


Figure 4.13. Average CRD percent loss cost vs. percent yield loss from 1980 to 2006

percentage of yield deviation from its trend yield, which is set at zero when there is no yield loss at all. Average is taken within a CRD. The straight line is the 45 degree line.

Ideally, there would be a one-to-one relationship between percent yield loss and percent loss cost. The figure indicates that for most CRD in most of the years, the proposed insurance policy is reasonably effective in terms of protecting corn yield losses. There are situations where payments were not triggered when yield losses occurred or payments were triggered even there was no yield loss. The reason is that the proposed insurance policy only insure against yield losses due to high temperature. There are other factors such as precipitation and pests affecting corn yield also. The impacts of factors other than temperature are likely large when the temperature is extremely high, which corresponds to larger deviation from the 45 degree line for cases of high percent yield losses shown in the figure. The nine points in the right-lower corner of Figure 4.13 correspond to the average CRD percent loss costs versus

percent yield losses in 1993 when it was an extremely wet year. It is clear that the percent yield losses are very high in every CRD while the percent loss costs are extremely low. Seven out of nine CRDs would not get any payment. Therefore, the proposed insurance contract would not be able to provide protection against risk of extreme moisture.

4.8 Discussion and conclusions

Weather index contracts have gained increased interest as new risk-management instruments in agricultural production. Although they solve a tremendous part of the problems the traditional crop insurance is plagued with, they also give rise to a new problem of basis risks. Much research has been done addressing the second layer or the technological basis risk. The first layer or spatial basis risk is often cited, yet rarely investigated. In this study, we apply sophisticated interpolation techniques to address the spatial basis risk of using temperature index to hedge against corn yield reduction in Iowa. Both the standard well-developed geo-statistical model and the less-developed Markov random field model are fitted to cooling degree data in Iowa. The comparisons of the prediction results indicate that MRF approach may be more promising in the sense of smaller cross-validation prediction mean squared error. To protect against yield reduction at farm-level, we propose a temperature index insurance policy which is essentially an exotic call option on ACDD and promises an indemnity payment to farmers when the underlying index ACDD is above the agreed strike value. Monte Carlo simulation technique is employed to calculate fair premium rate for the 99 county reference points in state of Iowa at various strike levels based on interpolated histories. The premium rates are not prohibitive, especially at high strike levels. Historical analysis shows that the proposed insurance policy successfully triggers indemnities

for places where the underlying temperature index ACDD is above the 26-year average. For extremely mild year such as 2004, no indemnity is triggered in any of the 99 county reference points. An analysis of the potential performance in year 1993 shows the limitation of the proposed index insurance policy. It will not provide protection against yield reduction due to reasons other than high temperature in the summer season. The historical analysis also shows that there is a systemic risk problem. When an indemnity payment is triggered at a location, chances are that indemnity payments are triggered in a large area around that location even the whole area of the state in hot years. Addressing spatial basis risk with a goal to provide insurance against yield reduction to farmers can not solve the supply-side problem of systemic risks. Given the high-level systemic nature of temperature, the proposed insurance policy is better supplied by large insurance companies or government agencies which can securitize the systemic risks via the Chicago Mercantile Exchange and over-the-counter security markets.

Data of various weather variables can be obtained from NOAA-NCDC. The MRF interpolation technique can be applied directly to interpolate weather indices for any location needed. Index insurance constructed based on interpolated index values should be more efficient in terms of protecting risk of insured and it is more important for situations where the underlying indices have a lower level degree of spatial correlation such as precipitation and hail.

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CHAPTER 5. GENERAL CONCLUSIONS

This dissertation presents research findings on three topics related to crop risk and insurance in the United States. The first topic estimates the willingness to pay for farm programs and crop insurance using prospect theory. The second topic develops a new method to measure the degree of risk poolability of an insurance book of business. The third topic shows how spatial basis risk from weather index insurance contracts can be reduced using Markiv random field models.

Chapter 2 examines the willingness to pay for 2002 farm bill programs and yield crop insurance and the programs' impacts on acreage decision of a representative Iowa farmer who receives both farm program benefits and crop insurance benefits. The mathematical problem of the representative farmer is to maximize expected profit and expected utility by allocating land to crops. The crops considered in this study are corn, soybeans and alfalfa hay. The results indicate that for farmers with reasonable risk aversion level, alfalfa hay will not enter into solution of acreage decision problems. This is not surprising because both corn and soybeans have higher expected crop profit than does hay. Furthermore, hay does not receive subsidies like corn and soybeans.

Instead of measuring producer welfare using expected utility to capture farmers' preferences over risky alternatives, we apply recent advances in decision theory and use prospect theory to measure welfare changes due to government programs. The combination of the value function of Kahneman and Tversky (1979) and the weighting function of Prelec (1998) provide the cumulative prospect theory framework which is shown to offer a globally more consistent utility functional form which can characterize farmers' preferences over risk.

Loss aversion and the weighting scheme are the reasons that cumulative prospect theory can better characterize a farmer's preference over risk alternatives than does expected utility theory. The implementation of cumulative prospect theory is demonstrated for a hypothetical mixed prospect with 5000 outcomes.

Welfare and decision analysis based on the calibration results of cumulative prospect theory are carried out for both harsh and no yield penalty for continuous cropping. The results lead to the following conclusions. First, there is no policy distortion to farmers' acreage decisions at either harsh or no yield penalties for planting corn after corn or soybeans after soybeans. Therefore, government programs act as lump-sum transfers to farmers with regard to their acreage decisions. Second, for the four existing government programs, farmers' willingness to pay per dollar of program cost is greatest for crop insurance. Given that farmers have crop insurance, the willingness to pay per dollar cost is much lower for loan deficiency payments, direct payments, and counter-cyclical payments. Last, efficiency measures of individual government programs are low because of three reasons. First, the sum of decision weights for the mixed prospect is 0.74, which implies that when we calculate the expected value of the mixed prospect, we eliminate 26% of the expected value at the very beginning. Farmer's willingness to pay thus the efficiency measure is lower accordingly. Second, the payment from each of the government programs only moves the farmer from a state of loss to another state of loss where loss aversion coefficient shrinks the WTP for programs by a factor of 1/2.25. The efficiency measure is lowered accordingly. And third, payments from these government programs are made not only when there is a loss but also when there is a gain. Expected value is essentially the weighted average of all values. Loss aversion leads to more value associated with a dollar increase in the loss part than that in the

gains part. In addition, the calibrated weighting scheme where β^+ is greater than β^- results in a higher decision weight for a loss than for a gain of the same order. Therefore, the change in expected value is lower when many of payments are made in the gains part relative to the change in expected value when all the payments are made in the loss part. An investigation showed that the proportion of payments made in the gains part for DPs, CCPs and LDPs are 43.2%, 23.5% and 24.7%, respectively. For CI, only 9.7% of the payments are made in the gains part, which account for the highest efficiency of CI among those government programs.

The endowment effect can be seen from the comparison of results of WTP scenario and WTA scenario since the high reference income point of WTA scenario is due to the endowment of all farm programs. The comparisons show that farmers value government programs more when he has a higher endowment. In another word, it leads to more pain to farmers to be deprived of those government programs when farmers originally have access to them than joy to farmers by providing farm programs when they are originally not available to farmers.

The construction of the cumulative prospect representations investigated in this study incorporates loss aversion and the value function is defined on deviations in wealth from a reference income level. The implementation and calibration show that the cumulative prospect representations investigated are able to reconcile significant small-scale risk aversion with reasonable degrees of large-scale risk aversion. Within the expected utility framework, turning down a modest-stake gamble means that the marginal utility of money must diminish very quickly for small changes in wealth. While prospect theory's value function has loss aversion, which means that turning down a modest-stake gamble does not necessarily imply that marginal utility of money diminishing quickly for small changes in

wealth. An empirical investigation shows that in addition to loss aversion, the choice of risk attitude level in cumulative prospect theory framework is also important. When the risk attitude is too low, people turn down small stakes also turn down large stakes because the convexity of the weighting function can not overcome the concavity of the utility function.

In Chapter 3, the focus shifts to the private insurance industry. The goal of this study is to develop a method to measure the risk poolability of any specific book of business quickly by simply knowing a few key statistics of the given book of business.

The method is developed using hail-insurance data. Exploratory analysis indicates that Western Iowa experiences greater hail losses than Eastern Iowa. Spatial statistics are employed to measure the maximum distance that hail losses independent. The results suggest that on average the range is 41 kilometers which means that on average hail losses are no longer correlated when two locations are more than 41 kilometers apart in Iowa. Based on this estimate of range, a measure called proportion index is created which measures the weighted proportion of pair distances exceeding the estimated spatial range. In addition, the well-known Herfindahl index is employed to measure the acreage concentration across customers of a book of business. To measure the portfolio risk, the coefficient of variation of total indemnities paid is employed which is equal to the standard deviation of the total indemnities divided by the mean of the total indemnities. A wide range of books of business is simulated applying Monte Carlo simulation technique. The simulated books of business vary by number of customers, location concentration and acreage concentration which are the key factors of a book of business affecting its risk poolability. Diversification measures and portfolio risk of each simulated book of business are calculated which provide the explanatory and explained variables of interest. After investigation, a Cobb-Douglas model is

obtained which captures the simulation procedure quite well. About 98% of variability of the risk poolability can be explained by the proportion index and Herfindahl index of the book of business.

Although it is derived based on hail loss cost data from the state of Iowa, the model is directly applicable to a larger geographic area, assuming similar pattern of spatial correlation, in the sense that both proportion index and Herfindahl index are unitless. For crop insurance or other lines of insurance in which risk is spatially related, we can still apply this method to derive a new model to predict risk poolability of a given book of business. Of course, investigation of the spatial correlation of the corresponding risk is needed in constructing the new models.

Chapter 4 turns to weather-based index contracts as alternative risk management instruments for crop losses. The goal of this study is to reduce the basis risk which is a major concern associated with developing weather-based index contracts. An application of the methodology is provided through an example of a temperature insurance contract designed to protect against farm-level corn yield reduction in Iowa. To address spatial basis risk, both the standard well-developed geo-statistical model and the less-developed Markov random field (MRF) model are fitted to the cooling degree data. The comparisons of the prediction results indicate that MRF approach may be more promising in the sense of smaller cross-validation prediction mean squared error. The technological basis risk is addressed through the use of indemnity factor which can be obtained through linear regression relating corn yield losses to the temperature index rises. The premium rates were calculated based on interpolated histories using Monte Carlo techniques. Historical analysis shows that the premium rates are

actuarially fair and the proposed insurance policy is effective in terms of protect corn yield losses due to high temperature.

Weather-caused yield losses are generally correlated across space due to the systemic nature of weather. Therefore, weather-based index contracts are better supplied by large insurance companies or government agencies which can aggregate crop losses to reduce the idiosyncratic risk in the aggregate portfolio and securitize the systemic risks via the Chicago Mercantile Exchange and over-the-counter security markets.

Data of various weather variables can be obtained from NOAA-NCDC. The MRF interpolation technique can be applied directly to interpolate weather indices for any location needed. Index insurance constructed based on interpolated index values should be more efficient in terms of protecting risk of insured and it is more important for situations where the underlying indices have a lower level degree of spatial correlation such as precipitation and hail.